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EFFECTS OF PHASE NOISE AND THERMAL NOISE UPON COHERENT PSK DEMO--ETC(U)

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EFFECTS OF PHASE NOISE AND THERMAL NOISE UPON COHERENT PSK DEMODULATION AND THEIR IMPACT ON PHASE NOISE SPECIFICATIONS FOR TERMINALS OF THE PHASE II DSCS

Prepared for

THE DEFENSE COMMUNICATIONS AGENCY

by

COMPUTER SCIENCES CORPORATION
6565 Arlington Boulevard
Falls Church, Virginia 22046

under

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EFFECTS OF PHASE NOISE AND THERMAL NOISE UPON COHERENT PSK DEMODULATION AND THEIR IMPACT ON PHASE NOISE SPECIFICATIONS FOR TERMINALS OF THE PHASE II DSCS

August 1974

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SUMMARY

Future communication systems such as the Phase II DSCS will use coherent phase-shift keyed (PSK) modulation with forward error control (FEC) coding and will be transmitted at X-band frequencies. Since coherent PSK systems using FEC are especially sensitive to signal spectral purity and since spectral purity is directly proportional to the transmission frequency, very careful system designs must be used in X-band transmission systems. These systems will require various frequency generation equipment such as atomic standards for long-term frequency accuracy, and various combinations of crystal oscillators, frequency synthesizers and frequency multipliers which are necessary to provide flexible transmit and receive frequency assignments.

The ultimate objective of this effort is to provide guidance on spectral purity requirements for the Phase II DSCS terminals and associated frequency generation equipment.

In this report we have limited ourselves to the following immediate objectives:

- Evaluate expected phase noise performance of various combinations of existing modulation and terminal subsystems operating in the Phase II DSCS.
- Use this insight to generate methods for specification of allowable phase noise as a function of desired system performance.

To reach these objectives it has been necessary to evaluate effects of phase noise upon (partially) coherent PSK demodulation performance and thereby gain insight into the dynamics of system performance. It is known that demodulation of coherent PSK signals requires knowledge of the phase of the original (unmodulated) carrier waveform. Estimates of carrier phase may be derived from the received signal by the well known techniques of phase-locked loop (PLL) theory. Here it is shown that optimum demodulation performance (i.e.,

minimum bit error rate (BER)) in the presence of phase noise and thermal noise is obtained by optimizing the bandwidth of the carrier tracking PLL. Using this technique, optimum demodulation performance for BPSK and QPSK systems is derived for terminals conforming to phase noise specifications designated "modified HT-MT" which is a modified version of an early DSCS HT-MT earth terminal incidental FM specification (SCA-2080A; see also Figure 4-1 which appears at the end of this summary). Results obtained also include the effects of rate 1/2, constraint length 7, convolutional encoding with 3 bit, soft decision Viterbi decoding.* Two other phase noise curves have also been synthesized, designated cesium II** and crystal II** (Figure 4-5), which are now considered to be realistic estimates of phase noise expected for terminal of the Phase II DSCS. Using these three types of terminal phase noise sources and convolutional encoding; an allowable interval of signaling rates are determined for BPSK and QPSK modulation systems when demodulation losses due to imperfect carrier tracking are limited to 0.2 dB. Results are summarized in Table S-1 (found at end of summary).

Subsystem Performance Evaluations

1. Using the "modified HT-MT" phase noise specification, Table S-1 shows that inadequate phase noise performance leads to both minimum and maximum signal rates even with the use of optimized phase estimators when using convolutional encoding and Viterbi decoding. However, with the most recent estimates of phase noise spectral densities (cesium II and crystal II) expected for the Phase II DSCS terminals, the upper bound on signaling rate is far greater than the data rates of interest.

*See note 1.

**A roman numeral II has been used here to help differentiate current data from that which appeared in a prior memorandum.

System performance has also been determined for three modems being designed for the Phase II DSCS using frequency converters conforming to the modified version of the incidental FM noise specification SCA-2080A (Figure 4-1), and the synthesized phase noise data of Section 4 (Figure 4-7).

2. Expected performance of the Radiation, Inc. BPSK modem (MD 921G) is summarized in Table S-2.

3. Results for the Raytheon, Inc. TDMA are summarized in Tables S-3(a) and (b) when the modem is operated with a 100 Hz one-sided PLL noise bandwidth and an optimized bandwidth, respectively. Dramatic improvements in system performance are noted here for a small increase in system complexity caused by the use of a variable bandwidth PLL.

4. Results for the Magnavox Research Laboratory, Inc. AN/USC-28 spread-spectrum modem are presented in detail in.[1] In the cited reference it is shown that the most critical performance requirements on carrier phase estimation performance occurred at the lowest information rates, where the phase noise of an improved AN/ASC-18 terminal would be similar to that of the synthesized phase noise (cesium II of Figure 4-7) expected for the HT-MT (AN/MSC-60) and the upgraded MSC-46 terminals. Therefore, at low data rates, demodulation performance for the USC-28 operating with the above DSCS terminals will be similar to that given in[1] when this modem is operated with the improved AN/ASC-18 terminal. At high frequency offsets from the carrier frequency, the synthesized phase noise curve (cesium II of Figure 4-7) will be better than that of the improved AN/ASC-18; therefore at high data rates performance of the USC-28 with the HT-MT or upgraded MSC-46 will be better than that shown in.[1]

Phase Noise Specification

A method of generating specification on oscillator phase noise has been devised based on phase noise power in a band specification. Analysis reveals that the shape of the oscillator phase noise spectral density is of secondary importance to the area under the phase noise spectral density curve in the region between the tracking filter 3-dB bandwidth (i.e., for a PLL this quantity is f_n) and the 3-dB bandwidth of the demodulator filter (for a matched filter this occurs at 1/2 the PSK symbol rate).

The design specifications on phase noise power in frequency bands as a function of demodulation losses for systems with rate 1/2, constraint length 7, convolutional encoding and 3 bit soft decision Viterbi decoding are summarized in Tables S-4, S-5 and Figure S-1.

Based on this method results are presented in Tables S-6 and S-7 which are the desired Phase II DSCS terminal phase noise specifications for the AN/MSC-60(HT) "Follow-on" and the AN/MSC-46 "Upgrade," respectively.

Finally, it should be pointed out that recent computer simulations on the performance of rate 1/2, constraint length 7, convolutional encoding with 3 bit soft decision Viterbi decoding performed at CSC indicates that the theoretical loss versus phase error variance functional derived in [2] and used in this report, may not be as severe as indicated. However, all of the phase noise specifications derived here are not unreasonable since they can be satisfied with state of the art techniques.

Table S-1. Allowable* Information Rates for Suppressed Carrier BPSK and QPSK Signaling With Several Possible Phase Noise Sources in the DSCS and With Soft Decision (3-bit), Rate 1/2, Constraint Length 7 Viterbi Decoding at BER = 10^{-5}

		Information Rate bps	Phase Noise Type ** Mod. HT-MT Phase Noise Specifications	Synthesized Data "Cesium"	Synthesized Data "Crystal"
BPSK	Power Loop	Minimum	450	< 75	< 75
		Maximum	$> 39 \times 10^6$	$> 39 \times 10^6$	$> 39 \times 10^6$
	Decision Fdbk	Minimum	300	< 75	< 75
		Maximum	$> 39 \times 10^6$	$> 39 \times 10^6$	$> 39 \times 10^6$
QPSK	Power Loop	Minimum	116×10^3	614×10^3	$< 4.8 \times 10^3$
		Maximum	20.6×10^6	$> 39 \times 10^6$	$> 39 \times 10^6$
	Decision Fdbk	Minimum	93.5×10^3	307×10^3	$< 4.8 \times 10^3$
		Maximum	20.6×10^6	$> 39 \times 10^6$	$> 39 \times 10^6$

*Acceptable loss $\leq .2$ dB with bandwidth optimized carrier tracking PLL

**Phase Noise contribution from 2 terminals and 1 equivalent satellite

Table S-2. Allowable* Information Rates for Radiation BPSK (Power Loop) Modem With Several Possible Phase Noise Sources in the DSCS and With Soft Decision (3-bit), Rate 1/2 Constraint Length 7 Viterbi Decoding at BER 10^{-5}

Information Rate bps	Phase Noise Type**
	Mod. HT-MT Phase Noise Specification or Synthesized "Cesium II" or "Crystal II"
Minimum	2.8×10^3
Maximum	$> 39 \times 10^6$

*Acceptable Loss $\leq$ 0.2 dB
**Phase Noise Contribution from 2 Terminals and 1 Equivalent Satellite

Table S-3(a). Allowable* Information Rates for Raytheon TDMA With PLL Noise Bandwidth of 100 Hz. Operation With Possible Phase II DSCS Noise Contributors and With Soft Decision (3-bit), Rate 1/2, Constraint Length 7, Viterbi Decoding at BER = 10^{-5}

	Information Rate Mbps	Phase Noise Type***		
		Mod. HT-MT Specification	Synthesized	
			"Cesium II"	"Crystal II"
BPSK	Minimum	1.2	1.2	1.2
	Maximum	> 38	> 39	> 39
QPSK	Minimum	**	~ 78	39
	Maximum	**	~ 78	> 78

*Acceptable Demodulation loss < 0.2 dB
**Losses always >> 0.2 dB
***Terminal phase contribution from 2 terminals and 1 equivalent satellite.

Table S-3(b). Allowable* Information Rates for Raytheon TDMA With Optimum PLL Noise Bandwidth. Operation With Possible Phase II DSCS Phase Noise Contributors and With Soft Decision (3-bit), Rate 1/2, Constraint Length 7, Viterbi Decoding at BER 10^{-5}

Information Rate Mbps		Phase Noise Type**		
		Mod. HT-MT Specification	Synthesized	
			"Cesium II"	"Crystal II"
BPSK	Minimum	0.307	< 0.15	< 0.07
	Maximum	> 39	> 39	> 78
QPSK	Minimum	2.4	~78	1.2
	Maximum	> 39	~78	> 78

*Acceptable Demodulation Loss < 0.2 dB

**Terminal phase contributions from 2 terminals and 1 equivalent satellite.

Table S-4. Phase Noise Specification Bands Assuming Worst Case, f^{-3}, Phase Noise Characteristics

Type of PSK Modulation	Data Bit Rate (R_{ib})	PSK Symbol Rate (R_{ms})	Phase Noise Specification Bands	
			DF Loop Implementation	Power Loop (MF) Implementation
	75 b/s	150 s/s	0.82 Hz - 75 Hz	0.68 Hz - 75 Hz
BPSK	19.2 kb/s	38.4 ks/s	0.2 kHz - 19.2 kHz	0.17 kHz - 19.2 kHz
M = 2	153 kb/s	307 ks/s	1.7 kHz - 153 kHz	1.4 kHz - 153 kHz
	1.2 Mb/s	2.4 Ms/s	13 kHz - 1.2 MHz	11 kHz - 1.2 MHz
	10 Mb/s	20 Ms/s	0.11 MHz - 10 MHz	91 kHz - 10 MHz
	19.2 kb/s	19.2 ks/s	6.7 Hz - 9.6 kHz	3.34 Hz - 9.6 kHz
QPSK	153 kb/s	153 ks/s	53 Hz - 76 kHz	26 Hz - 76 kHz
M = 4	1.2 Mb/s	1.2 Ms/s	0.41 kHz - 0.6 MHz	0.21 kHz - 0.6 MHz
	10 Mb/s	10 Ms/s	3.5 kHz - 5 MHz	1.7 kHz - 5 MHz

Specification Band for TDMA Operation: 23 Hz to 40 MHz

Table S-5. Equivalent Power Loss and Corresponding Phase Noise Variances
Based on Conservative f^{-3} Phase Noise Characteristic Design and Gaussian Loss Approximation

Type of Modulation (M)	Equivalent Power Loss (L_{tot} dB)	Total Amount of Phase Variance (σ_{tot}^2 dB)	Loop Variance (σ_{th}^2 dB)	Phase Error Variance due to Phase Noise	
				1 of 2 Terminals (dB)	1 of 3 Terminals (dB)
BPSK M = 2	0.1	-18.0	-19.8	-25.8	-27.5
	0.2	-15.0	-16.8	-22.8	-24.5
	0.3	-13.3	-15.1	-21.1	-22.8
	0.4	-12.7	-14.5	-20.5	-22.2
QPSK M = 4	0.1	-31.0	-32.8	-38.8	-40.5
	0.2	-28.0	-29.8	-35.8	-37.5
	0.3	-26.0	-27.8	-33.8	-35.5
	0.4	-25.0	-26.8	-32.8	-34.5
	0.5	-24.0	-25.8	-31.8	-33.5

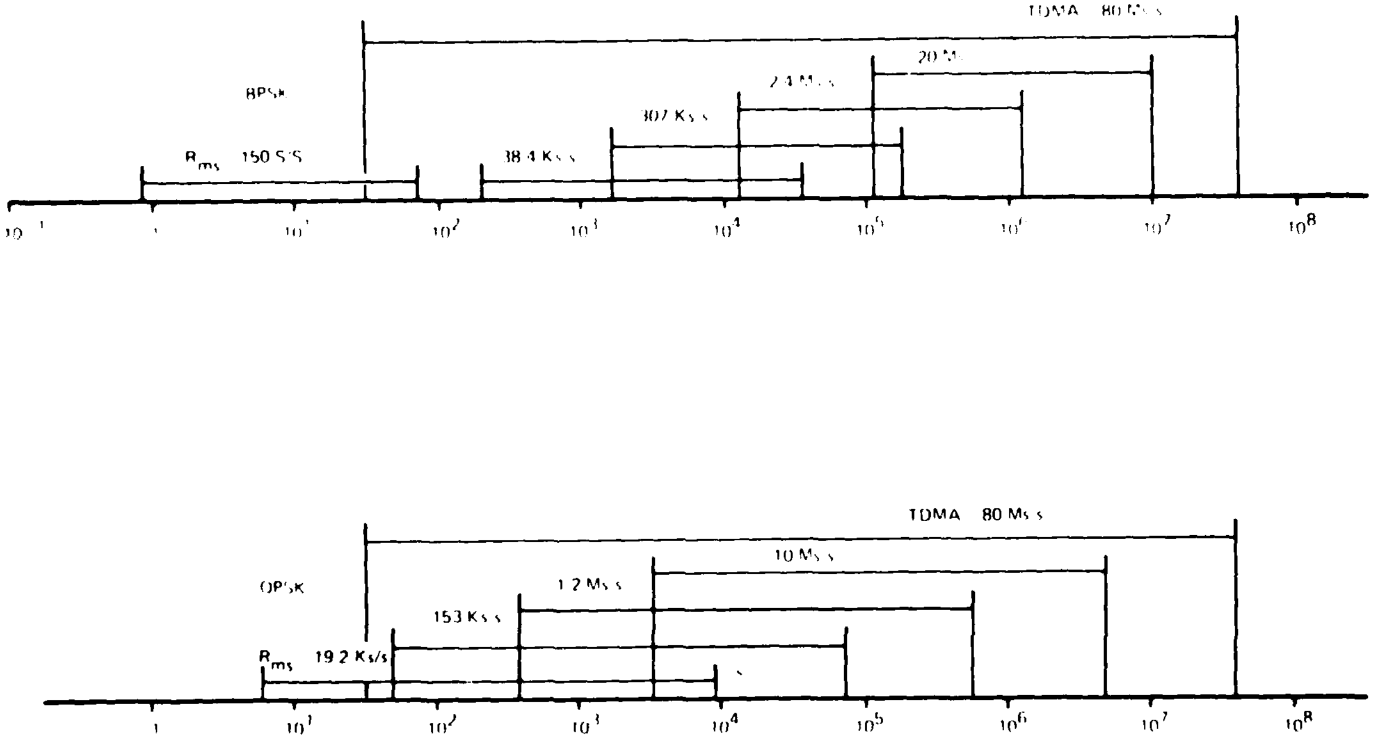


Figure S-1. Phase Noise Specification Bands (σ_{th}^2 -17 dB for BPSK and σ_{th}^2 -30 dB for QPSK; DF Loop Implementation)

Table S-6. Proposed Specification on Spectral Purity for the Follow-on AN/MSC-60 (HT)

1.0 Spectral Purity

The total spurious content added to any transmitted or received carrier, including phase noise and discrete spurious signals, shall not exceed conditions specified in the following paragraphs.

1.1 Spectral Purity for BPSK-QPSK

a. Total spurious content from both sides of the carrier at least 25 dB below the carrier level when measured in a band 0.6 Hz to 75 Hz from the carrier frequency.

b. Total spurious content from both sides of the carrier at least 37.5 dB below the carrier level when measured in the following frequency bands:

(b-1) 5 Hz to 16 kHz from the carrier frequency

(b-2) 20 Hz to 76 kHz from the carrier frequency

(b-3) 200 Hz to 0.6 MHz from the carrier frequency

(b-4) 1.7 kHz to 5 MHz from the carrier frequency

(b-5) 7 kHz to 20 MHz from the carrier frequency

1.2 Spectral Purity for FDM FM

Total spurious content from both sides of the carrier measured in any 3 kHz bandwidth shall be below the carrier level as follows:

a. 57 dB minimum from 12 kHz to 20 kHz

b. 62 dB minimum from 20 kHz to 30 kHz

c. 65 dB minimum from 30 kHz to 300 kHz

1.3 Spectral Purity for Burst Coherent TDMA

Total spurious content from both sides of the carrier shall be at least 37.5 dB below the carrier level when measured in a band 23 Hz - 40 MHz from the carrier frequency.

Table S-7. Proposed Specification* on Spectral Purity for the AN/MSC-46 "Upgrade" Terminal

1.0 Spectral Purity

The total spurious content added to any transmitted or received carrier, including phase noise and discrete spurious signals, shall not exceed conditions specified in the following paragraphs.

1.0 Spectral Purity for BPSK-QPSK

a. Total spurious content from both sides of the carrier at least 37 dB below the carrier level when measured in the following frequency bands:

(a-1) 0.6 Hz to 75 Hz from the carrier frequency

(a-2) 1.8 Hz to 200 Hz from the carrier frequency

b. Total spurious content from both sides of the carrier at least 37.5 dB below the carrier level when measured in the following frequency bands:

(b-1) 5 Hz to 16 kHz from the carrier frequency

(b-2) 20 Hz to 76 kHz from the carrier frequency

(b-3) 200 Hz to 0.6 MHz from the carrier frequency

(b-4) 1.7 kHz to 5 MHz from the carrier frequency

(b-5) 7 kHz to 20 MHz from the carrier frequency

1.2 Spectral Purity for FDM/FM

Total spurious content from both sides of the carrier measured in any 3 kHz bandwidth shall be below the carrier level as follows:

Table S-7. Proposed Specification* on Spectral Purity for the AN/MSC-46 "Upgrade" Terminal (Cont'd)

a. 57 dB minimum from 12 kHz to 20 kHz b. 62 dB minimum from 20 kHz to 30 kHz c. 65 dB minimum from 30 kHz to 300 kHz 1.3 <u>Spectral Purity for Burst Coherent TDMA</u> Total spurious content from both sides of the carrier shall be at least 37.5 dB below the carrier level when measured in a band 23 Hz - 40 MHz from the carrier frequency.

*Specifications do not include effects of reference standard. Also assumes that terminal phase noise is dominated by reference at frequencies below 200 Hz.

SECTION 1 - INTRODUCTION

The need for specifications on phase noise arise because all information conveyed in a coherent PSK signal resides in phase changes added to an unmodulated phase reference (carrier phase reference). The phase reference is, alas, always imperfect even at the transmitter since it always contains noise perturbations[3,4] characterized as phase noise for high frequency perturbations and long-term drifts for low frequency perturbations. Because the receiver has no a priori knowledge of these phase perturbations in time, the receiver has to distinguish between the PSK modulation and the phase noise. Our problem at the receiver then becomes one of estimating carrier phase perturbations in the presence of PSK modulation and additive white Gaussian noise (AWGN).[5,6]

A perfect reference is, by definition, physically unrealizable. Non-realizability occurs because the parameters which characterize the reference are not truly constant with time but have random noise perturbations superimposed. Optimum performance in PSK systems demands that we estimate phase noise fluctuations of the reference phase so that their effects can be minimized. Since all information is contained in the PSK signal phase, amplitude noise effects are only of significance when passed through devices which cause amplitude noise to be converted to phase noise.

Estimation accuracy may be characterized in the mean square error (MSE) sense by the total phase estimation error variance $\sigma^2_{tot.}$ The total error variance is the sum of two terms: (1) a phase error variance due to the effects of thermal noise σ^2_{th}, and (2) a phase error variance σ^2_{pn} due to the inability of the carrier phase estimator to completely estimate the entire phase noise process on the received signal.

It is known that the phase error variance due to thermal noise is directly proportional to the noise bandwidth of the reference phase estimator.[5] Here we show that the error variance due to phase noise is inversely proportional to various functions of the phase estimator noise bandwidth. (These inverse functions are directly related to the phase noise spectrum present on the reference signal.) Thus, a set of opposing constraints is given for minimizing phase error variance resulting in an optimum phase estimator noise bandwidth (optimum in the sense that it provides the minimum mean square error (MMSE)).

In this report considerable effort is directed towards derivation of this optimum bandwidth, and thus the MMSE for systems using second order PLLs of the power variety (squaring, quadrupling, etc.) or decision-directed feedback type, or pure second order PLLs when an auxiliary unmodulated carrier sinusoid is utilized. Since the order of an optimal linear phase estimator is a function of the order of the oscillator phase noise spectral density, higher order PLLs (i.e., 3rd or 4th) may be desirable in certain instances, however, the analysis could easily be extended with some additional algebraic complexity.

In the preceding we have focused attention on the fidelity of the carrier phase estimator as expressed by its total phase error variance σ^2_{tot} and we have only hinted that this parameter is directly related to a demodulation performance in a coherent PSK system. In the literature several analyses are available,[2,7,8] which show demodulation loss from ideal performance in a coherent PSK system when using a noisy phase reference. These analyses account only for thermal noise corruption of the carrier phase estimator. Here the loss functionals derived in these references are extended to include the additional degradations caused by incomplete estimation of the phase noise process on the received signal.

Furthermore, future communication systems (e.g., Phase II DSCS) will be increasingly sensitive to errors in carrier phase estimation due to the use of forward error control coding (FEC). Because of the increased sensitivity

of PSK systems with FEC, it is of utmost importance to: (1) Obtain estimates of all phase noise added to any coherent PSK transmission system, (2) Calculate the exact performance (expressed in the sense of mean square error (MSE) by the phase estimation error variance σ^2) of various carrier phase estimation techniques in the presence of thermal noise (AWGN) and phase noise, (3) Apply the phase estimation error variance to various decoding loss functions [2,7,8] and thereby obtain demodulation loss curves, and finally (4) Derive specifications on adequate phase noise performance for transmission and receive facilities (of the Phase II DSCS).

The use of FEC is suppressed carrier systems allows signaling at extremely low energy per bit to noise density ratios, thus carrier phase estimates must have additional signal processing gain to provide immunity against a relatively large amount of AWGN. This leads to requirements for extremely small bandwidths for carrier phase estimation and therefore places additional restrictions on the allowable level of phase noise.

The problem of estimating coherent PSK system performance in the presence of thermal noise and phase noise may be formulated more precisely by the following mathematical representation. A typical receiver signal in a suppressed carrier coherent PSK modulation system is:

$$r(t) = V(t) + n(t)$$

where

$$V(t) = \left|V_o + \epsilon(t)\right| \sin\left|2\pi\gamma_o t + \frac{2\pi k}{m} + \delta\phi(t) + \theta\right|$$

and

$n(t)$ = an additive white gaussian noise (AWGN)

V_o = the nominal amplitude

γ_o = the nominal frequency

m	=	the maximum number of phase positions (e.g., $m = 4$ QPSK)
k	=	$0, 1, , , m-1$ determines the modulation angle in the interval $[t, t+T]$
ψ	=	an arbitrary but fixed phase offset
$\epsilon(t)$	=	an amplitude noise fluctuation
$\delta\phi(t)$	=	the phase noise fluctuation including all amplitude fluctuations which have been passed through AM to PM conversion devices.

Assuming that $\frac{\epsilon(t)}{V_o} \ll 1$, amplitude fluctuations can be ignored. Since the constant angle ψ is either known or can be estimated, its effects may be ignored. If the estimate of the phase noise term $\delta\hat{\phi}(t)$ can be made accurately $\{$i.e., $|\delta\hat{\phi}(t) - \delta\phi(t)| << \delta\phi(t)\}$, then the effects of phase noise can be minimized. Of course carrier phase estimates $\delta\hat{\phi}(t)$ will be less than perfect since they must be made in the presence of AWGN and, in the case of suppressed carrier system, simultaneously in the presence of phase modulation. Errors in carrier phase estimates which are induced by AWGN can be minimized by using an estimator with long averaging time (small bandwidth). However, if the phase fluctuations $\delta\phi(t)$ contain high frequency spectral components with high energy content, a phase estimator with short averaging time (large bandwidth) is required leading to a conflicting set of constraints and an optimum averaging time (bandwidth) for optimum performance.

SECTION 2 - SYSTEM DEFINITIONS, PHASE NOISE TERMINOLOGY AND THE IMPACT OF ANY COLOR PHASE NOISE ON PARTIALLY COHERENT PSK SYSTEM

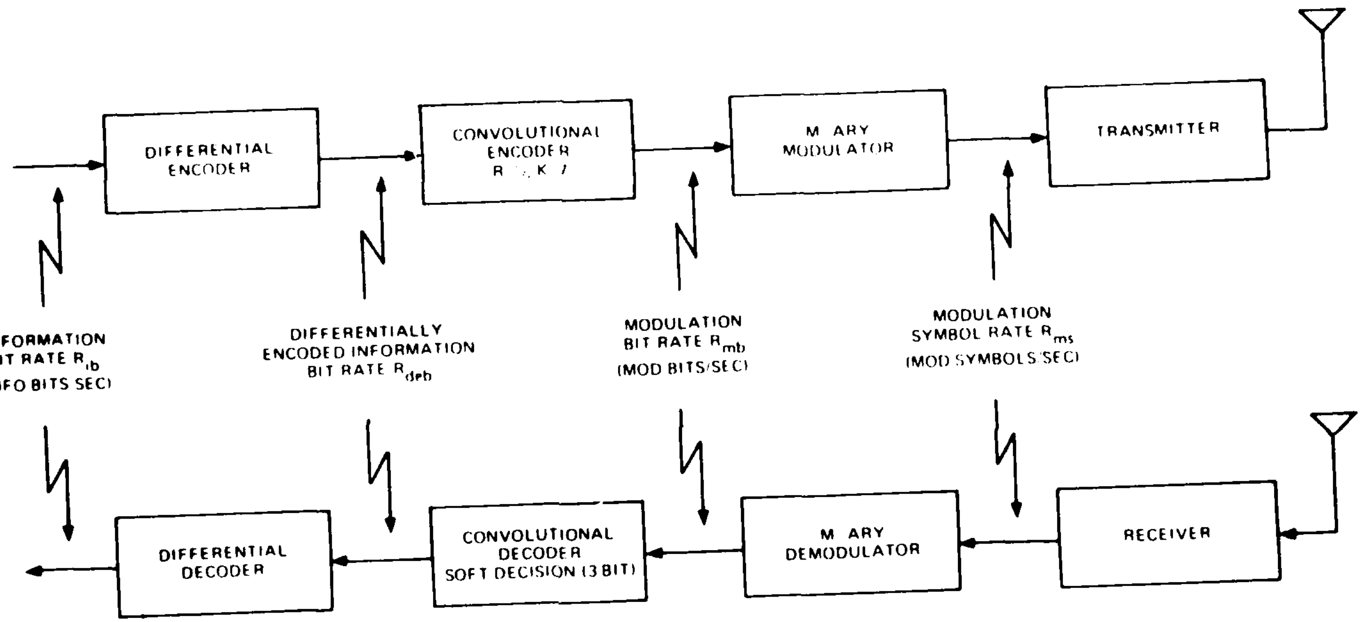
2.1 SYSTEM DEFINITIONS

In a complex satellite communication system such as the DSCS which uses convolutional (rate 1/2) and differential encoding together with M-ary ($M = 2, 4$) PSK modulation, a common source of confusion is the terminology used by different people to describe the same phenomenon. One designer's bits become another designer's symbols especially for people concerned with coding and modulation. Because the arguments for naming these items are extremely convincing, depending upon the designer's area of expertise, the approach used here will be to define symbols via a system diagram and let the reader change the names to suit his requirements.

Figure 2-1 depicts the general system diagram. Since the main item of interest here is the modulation-demodulation system, the term modulation bits at rate R_{mb} is used to describe the *input transition* rate to the M-ary modulator which then produces modulation symbols at rate R_{ms}. Henceforth, unless otherwise stated, all references to symbols or symbol rate refer to modulation symbols and all references to bits refer to modulation bits as described above.

2.2 PHASE NOISE TERMINOLOGY

Another source of confusion may arise from the specification of oscillator phase noise spectral density. In this memorandum the definition which has been used is a 1-sided spectrum at low pass (at baseband) as defined by the symbol $S_{\delta\phi}(f)$ and in Figure 2-2(a). Other possible representations of phase noise spectral density are given by Figures 2-2(b, c, d). Many hardware developers choose to display phase noise spectral density by plotting only the upper sideband of Figure 2-2(d). The ordinate is then referred to as single-side band noise to carrier ratio and sometimes denoted as $\mathcal{L}(f)$. No problems arise as long as it is clear which spectral density representation is being used.[3], [4]



$$\left\{ \begin{array}{l} R_{ms} = R_{mb}/\log_2 M \\ R_{mb} = 2R_{ib} \end{array} \right\}$$

Figure 2-1. M-ary PSK System

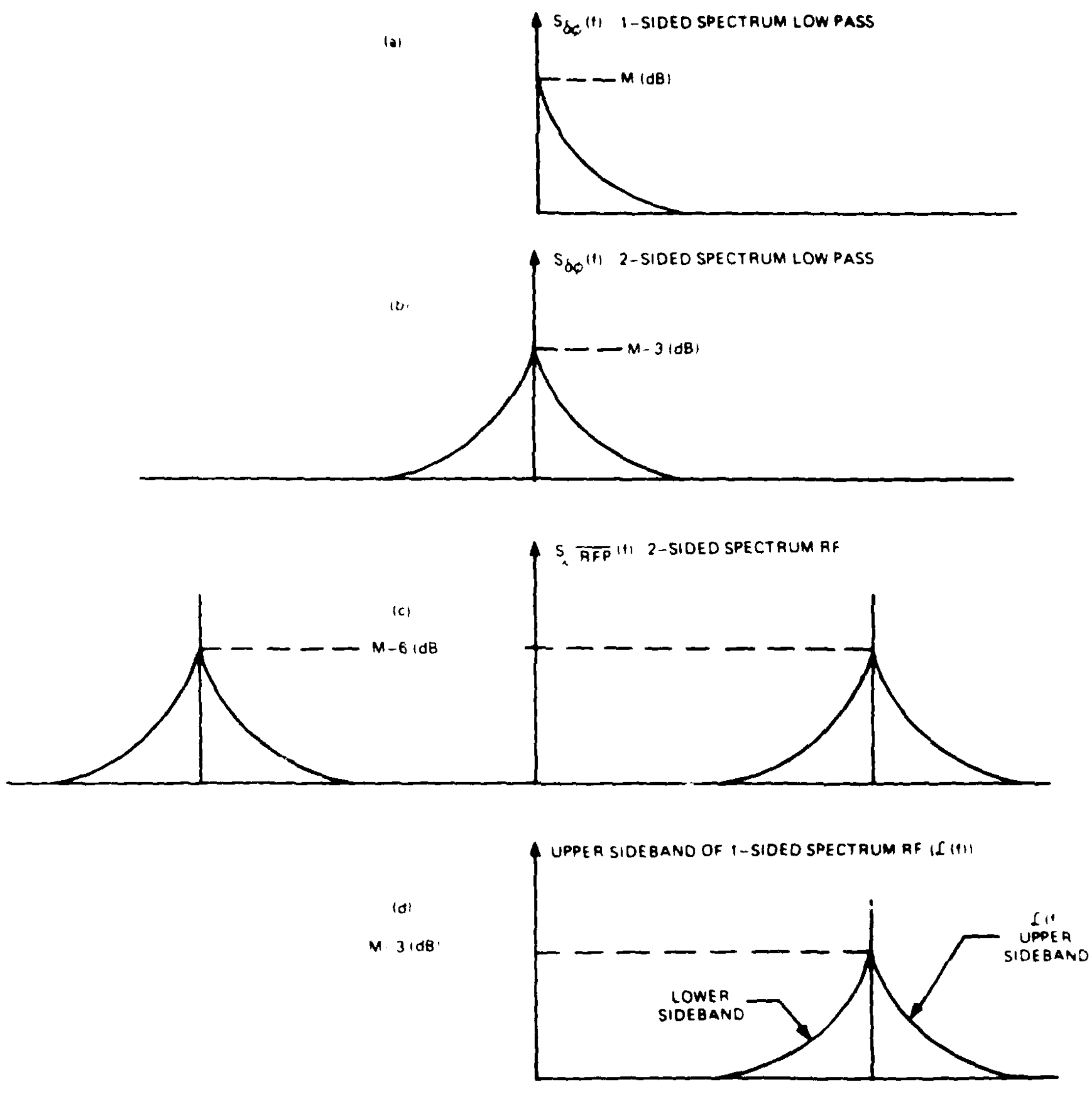


Figure 2-2. Phase Noise Density; Definitions Adopted in This Report

2.3 GENERAL IMPACT OF ANY COLOR PHASE NOISE ON PARTIALLY COHERENT PSK SYSTEMS

The introduction of this report has indicated that a demodulation performance tradeoff exists between a design which efficiently tracks out phase noise perturbations present on the transmitted PSK signal and a design that excludes as much thermal noise as possible. A common misunderstanding in the design and description of coherent PSK communications is that the effects of a white oscillator phase noise process on the transmitted signal may somehow be treated as an equivalent additive thermal noise at the receiver front end. The distinction between these two phenomena may be seen from the following arguments. Phase noise results from multiplicative processes which cause a pure rotation of the phase reference relative to the PSK decision region structure. Since a rotation affects decisions on any transmitted PSK symbol in precisely the same manner, i.e., independent of the symbcl phase, it is possible by estimating the angle of rotation of the phase reference to compensate for phase noise effects. The phase component caused by the additive, thermal, noise affects the signal in a different manner. That is, a particular thermal noise waveform will cause a phase rotation the magnitude and direction of which is dependent upon the received PSK symbol phase. Therefore, phase rotation of the reference caused by thermal noise cannot be removed in a way that is independent of the receiver symbol sequence as in the case for carrier phase noise.

Since the ultimate aim is to coherently detect the transmitted PSK modulation angle (data symbols) with as few errors as possible, it is desirable to maximize tracking of the carrier phase noise process (including the flat (white)* portion of the spectrum) simultaneously excluding as much additive thermal noise as possible.

*Of course if the entire oscillator phase noise spectrum is flat (white) and high level, the discussion of coherent system is absurd.

It is shown in Section 3 and in [6] that the demodulation process usually involves a matched filter or integrate-and-dump filter which suppresses the effects of errors in the carrier phase reference at frequencies beyond one-half of the PSK symbol rate. Thus, the most desirable carrier phase tracking system should track as much of the phase noise process (including the flat portion of the phase noise spectral density) as possible, within a bandwidth equal to one-half of the symbol rate, simultaneously excluding as much additive thermal noise as possible.

It is tacitly assumed that when designing a coherent PSK system the design is ultimately limited by thermal noise effects on the phase reference rather than by phase noise. That is, by increasing the carrier tracking system bandwidth, in an attempt to track all the desired carrier phase information, additional additive thermal noise enters the system degrading the coherence of the carrier phase reference and ultimately increasing the total demodulation loss. Thus, a trade-off in carrier tracking bandwidth may be established which minimizes the total demodulation loss due to the untracked portion of carrier phase noise and that due to thermal noise corruption of the carrier phase reference. The preceding results are derived analytically in Section 3.

SECTION 3 – EFFECTS OF ADDITIVE GAUSSIAN NOISE AND PHASE NOISE UPON COHERENT PSK DEMODULATION

3.1 ANALYTICAL STUDIES AND TRADEOFF ANALYSES

Multiplicative noise on the received signal and its residual effects upon (partially) Coherent PSK signal demodulation are investigated in this section. The multiplicative noise or phase noise originates from frequency converters (mixers) in which the signal is multiplied with another signal containing either phase noise or additive noise which causes phase noise as the signals are multiplied. Another source is AM/PM conversion that is produced by certain system components, e.g., TWTs. The phase noise process generally includes both random and deterministic components (spurious signals).

The statistical information about phase noise is generally limited to the second order statistics, i.e., the phase noise process is specified by its power spectral density. By using δ-function formalism one can also include the spurious components in the density spectrum. Based on physical characteristics of signal oscillators [3] the power spectral density $S_{\delta\varphi}(f)$ of the phase noise process $\varphi(t)$ is of the form

$$S_{\delta\varphi}(f) = h_0 + \frac{h_1}{f} + \frac{h_2}{f^2} + \frac{h_3}{f^3}; \quad \text{continuous spectrum}$$

$$+ \sum_{k=1}^{N} \frac{\beta_k^2}{2}\, \delta(f - f_k) \quad ; \quad \text{discrete spectrum}$$

Here $S_{\delta\varphi}(f)$ is defined as the one-sided ($f > 0$) spectrum that would be obtained if the oscillator output signal was coherently demodulated (translated to baseband) by a perfect reference signal. The first four terms containing values of $\{h_k\}$ specify the continuous spectrum, while the $\{\beta_k^2/2\}$ are the powers of spurious signals relative to the total signal power at the offset frequencies f_k.

However, in many cases the output signal from an oscillator is filtered to reduce the phase noise power thereby modifying the spectral representation.

We are not directly interested in the phase noise sources but rather the resulting phase noise (multiplicative noise) present in the received signal influencing the PSK symbol demodulation. In general, the phase noise process of the received signal will have a power spectral density with spectral components given in Equation (3-1). Therefore, the spectral density $S_{\delta\phi}(f)$ of the phase noise process $\phi(t)$ at the receiver input lends itself to the determination of the influence or degradation of the PSK demodulation performance since from it the phase error variance at the point of the symbol decision can be obtained. With this variance at hand we can determine the equivalent power loss caused by the phase noise in accordance with [2], [6]-[8].

First, assume that PSK demodulation is performed with a carrier reference which is not corrupted by thermal noise but does not contain information about the phase noise process on the received PSK signal. In this case the phase noise variance σ^2_{pn} due to phase noise at the symbol decision point is given by

$$\sigma^2_{pn} = \int_0^\infty S_{\delta\phi}(f) \left| M(f) \right|^2 df \qquad (3\text{-}2)$$

where $M(f)$ is the frequency transfer function of the PSK demodulator (usually a matched filter). Equation (3-2) is an approximation that is valid when the amplitude of the phase noise process $\phi(t)$ is small. A few simple relationships show how Equation (3-2) is derived. For an arbitrary phase angle θ we have

$$\begin{aligned} e^{i[\theta + \phi(t)]} &= e^{i\theta} \cdot e^{i\phi(t)} \\ &\cong e^{i\theta}[1 + i\phi(t)] \qquad (3\text{-}3) \end{aligned}$$

provided $\phi(t) << 1$. With the impulse responses $m(t)$ of the filter $M(f)$ being normalized so that $\int m(t)\, dt = 1$, i.e., $M(0) = 1$, the output of the detection filter

$$\int m(t-\tau)\, e^{i[\theta + \phi(\tau)]}\, d\tau$$

$$\cong e^{i\theta}[1 + i \int m(t-\tau)\, \phi(\tau)\, d\tau]$$

$$\cong \exp\{i[\theta + \int m(t-\tau)\, \phi(\tau)\, d\tau]\} \tag{3-4}$$

since $\phi(t) << 1$ also implies that $\int m(t-\tau)\,\phi(\tau)\, d\tau << 1$. This shows that the demodulation filter acts as a linear filter on the phase process $\phi(t)$, provided the amplitude of $\phi(t)$ is small.

A common receiver implementation uses an integrate and dump circuit as a detection filter. The integration operation

$$\frac{1}{T_s} \int_0^{T_s} (\cdot)\, dt \tag{3-5}$$

over the modulation symbol period T_s corresponds to the filter characteristic

$$M(f) = \frac{\sin \pi f T_s}{\pi f T_s} \cdot e^{i \pi f T_s} \tag{3-6}$$

With this detection filter Equation (3-2) takes the form

$$\sigma^2_{pn} = \int_0^\infty S_{\delta\phi}(f) \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df \tag{3-7}$$

An attempt to evaluate this integral with $S_{\delta\phi}(f)$ according to Equation (~~2~~3-1) will yield an unbounded variance σ^2_{pn} unless $h_1 = h_2 = h_3 = 0$. Since at least one of these parameters will not vanish in a real system application, a system using a carrier reference which does not contain information about the phase noise process on the received PSK signal is impossible. It is obvious, however, that noise would have no effect if the carrier reference signal tracked the phase noise perfectly to remove its influence.

A phase-locked loop implementation of the carrier reference signal will track slow changes in the received carrier phase and will therefore at least

partially track the phase noise process. Increased tracking ability is obtained by increasing the phase-locked loop bandwidth. However, this will make the phase estimate more noisy due to less filtering of the additive channel noise. Therefore, a trade-off between phase noise tracking and filtering of additive channel noise is required to determine the optimum phase-locked loop bandwidth that will yield minimum performance degradation in the PSK demodulation process. To perform this trade-off analysis we have to consider the particular frequency characteristic of the phase-locked loop as well as its resulting noise bandwidth, $B_{\odot}$.

Now given the closed-loop phase-locked loop transfer function H(f), the spectral densities at various points of the phase-locked loop and demodulator circuits can be determined (see Figure 3-1). The spectrum associated with the input phase noise process at various points is obtained by multiplying $S_{\delta\phi}(f)$ by the absolute square of the frequency transfer function to the specific point of interest. In particular, the phase noise spectrum at the input to the symbol demodulator filter is given by

$$S_{\delta\phi}(f)\,|\,1 - H(f)\,|^2 \tag{3-8}$$

The additive Gaussian noise will also cause phase noise via the phase-locked loop. Its spectrum at the demodulator filter input is[3]

$$\frac{N_o}{E_s R_s}\,|\,H(f)\,|^2 \quad (f > 0) \tag{3-9}$$

where $E_s R_s$ equals the received carrier power. Thus at the mixer output before the demodulation filter we have the total phase noise density

$$S_{\delta\phi}(f)\,|\,1 - H(f)\,|^2 + \frac{N_o}{E_s R_s}\,|\,H(f)\,|^2 \tag{3-10}$$

This implies that the total phase noise variance at the output of the demodulator

partially track the phase noise process. Increased tracking ability is obtained by increasing the phase-locked loop bandwidth. However, this will make the phase estimate more noisy due to less filtering of the additive channel noise. Therefore, a trade-off between phase noise tracking and filtering of additive channel noise is required to determine the optimum phase-locked loop bandwidth that will yield minimum performance degradation in the PSK demodulation process. To perform this trade-off analysis we have to consider the particular frequency characteristic of the phase-locked loop as well as its resulting noise bandwidth, B_{φ}.

Now given the closed-loop phase-locked loop transfer function H(f), the spectral densities at various points of the phase-locked loop and demodulator circuits can be determined (see Figure 3-1). The spectrum associated with the input phase noise process at various points is obtained by multiplying $S_{\delta\varphi}(f)$ by the absolute square of the frequency transfer function to the specific point of interest. In particular, the phase noise spectrum at the input to the symbol demodulator filter is given by

$$S_{\delta\varphi}(f)|1 - H(f)|^2 \tag{3-8}$$

The additive Gaussian noise will also cause phase noise via the phase-locked loop. Its spectrum at the demodulator filter input is[3]

$$\frac{N_o}{E_s R_s} |H(f)|^2 \quad (f > 0) \tag{3-9}$$

where $E_s R_s$ equals the received carrier power. Thus at the mixer output before the demodulation filter we have the total phase noise density

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This implies that the total phase noise variance at the output of the demodulator

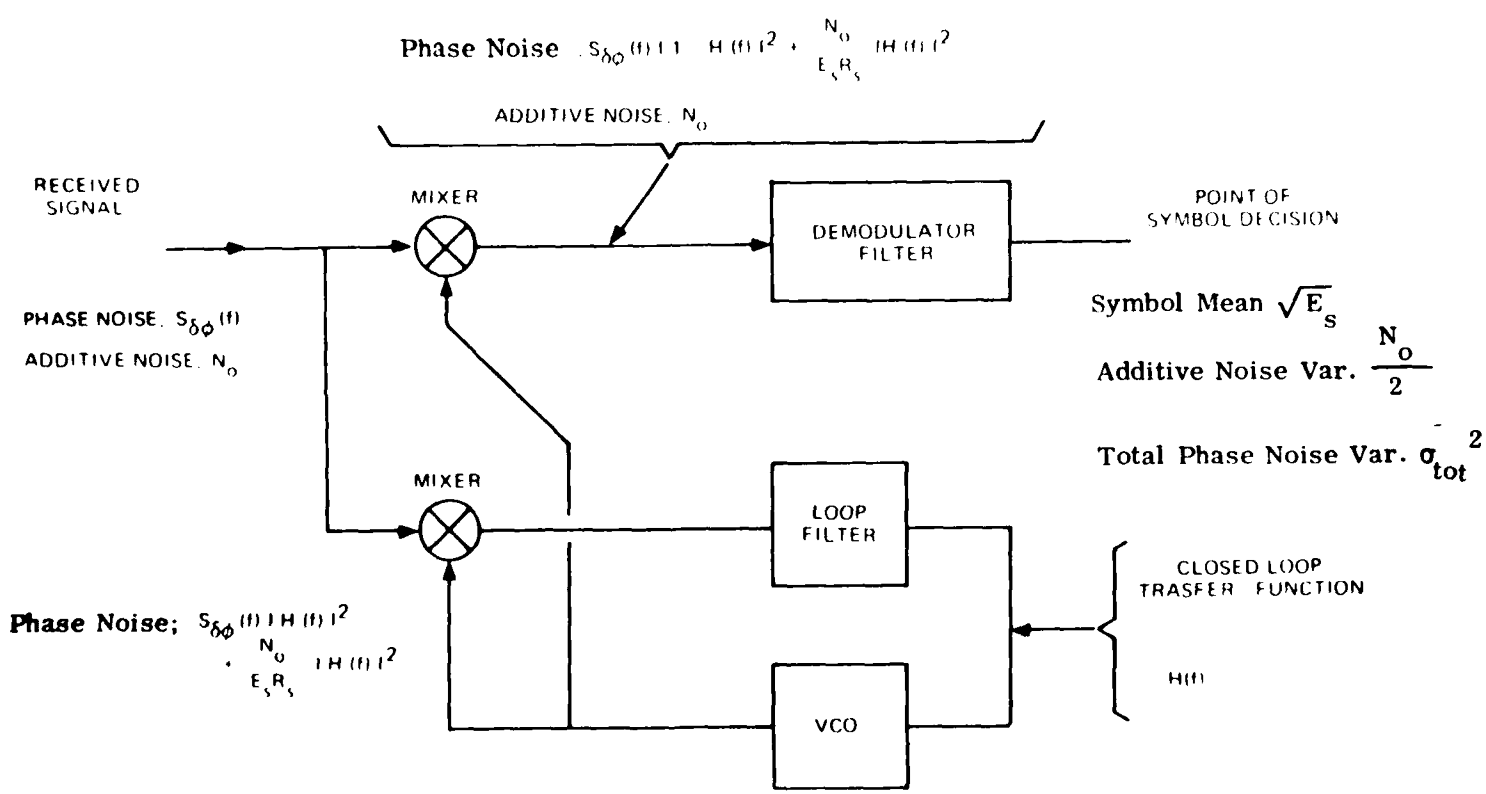

Figure 3-1. Spectral Densities at Various Points of a (Partially) Coherent PSK Receiver

filter, an integrate and dump filter, is given by

$$\sigma^2_{tot} = \sigma^2_{pn} + \sigma^2_{th} \tag{3-10a}$$

where

$$\sigma^2_{pn} = \int_0^\infty S_{\delta\phi}(f)\,|1 - H(f)|^2 \left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2 df \tag{3-10b}$$

and

$$\sigma^2_{th} = \int_0^\infty \frac{N_o}{E_s R_s}\,|H(f)|^2 \left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2 df \tag{3-10c}$$

Figure 3-2 shows the general characteristics of $|H(f)|^2$ and $|1 - H(f)|^2$ and indicates that the effect of $S_{\delta\phi}(f)$ is reduced for low frequencies since $|1 - H(f)|^2$ approaches zero for decreasing frequencies. In other words, the phase-locked loop partially tracks the low frequency components of the phase noise process. The more the low frequency region is suppressed by $|1 - H(f)|^2$, the less the phase noise variance σ^2_{pn} resulting from the phase noise process $\phi(t)$. On the other hand, this increase will make the variance σ^2_{th} larger since the area under $|H(f)|^2$ will be larger. Therefore, to minimize the total variance σ^2_{tot}, the closed-loop filter characteristic $H(f)$ should be judiciously chosen.

In general, $S_{\delta\phi}(f)$ will contain the f^{-3} component [i.e., $h_3 > 0$ in Equation (3-1)] that suggests that $|1 - H(f)|^2$ should approach at least as fast as f^3. This requires a second- or higher-order phase-locked loop implementation. Considering that we know only that the phase noise spectrum $S_{\delta\phi}(f)$ is dominated by a spectrum of the form in Equation (3-1), a good system solution is given by a filter that makes $|1 - H(f)|^2$ maximally flat at $f = 0$; "Butterworth filter."

A second-order maximally flat PLL filter defines

$$|1 - H(f)|^2 = \frac{f^4}{f_n^4 + f^4} \tag{3-11}$$

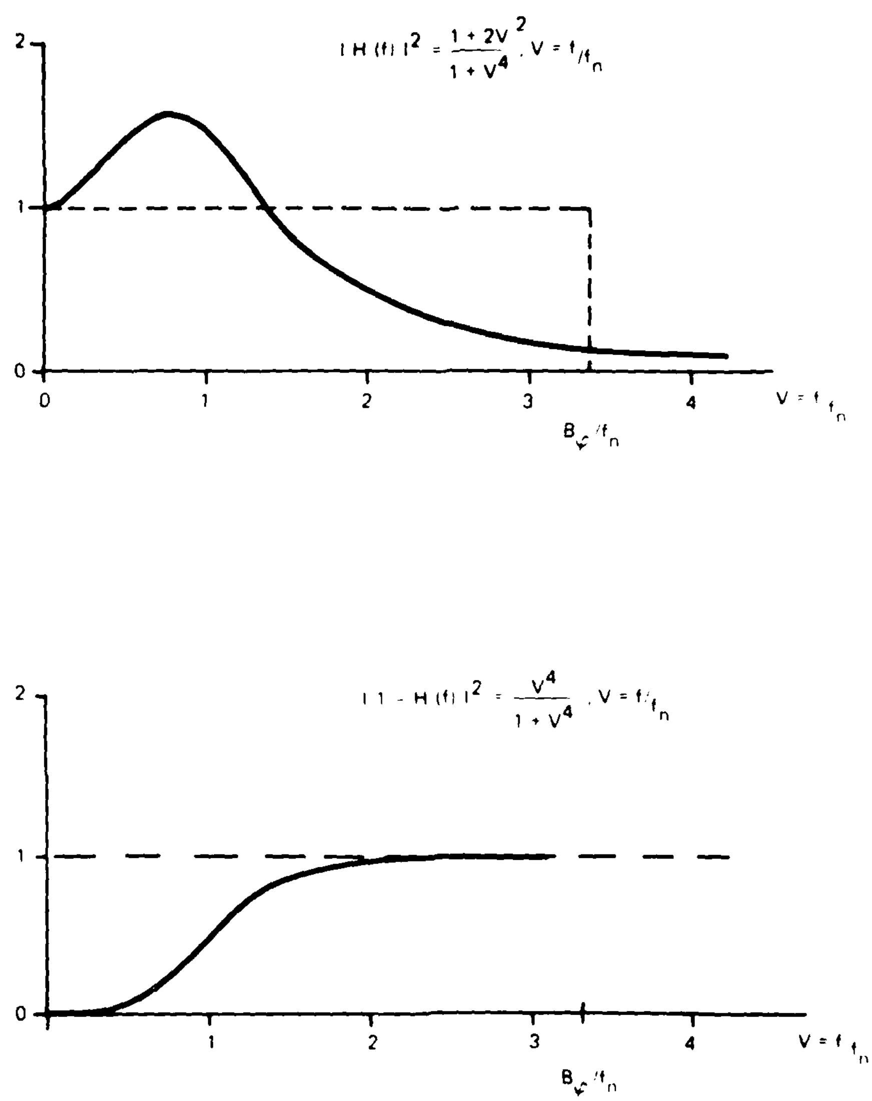

Figure 3-2. Frequency Characteristics of a Second-Order Phase-Locked Loop with Damping Factor = 0.707

which is consistent with the characteristic

$$H(f) = \frac{f_n^2 + i\sqrt{2}\, f_n f}{f_n^2 + i\sqrt{2}\, f_n f - f^2} \tag{3-12}$$

making

$$|H(f)|^2 = \frac{f_n^2 (f_n^2 + 2 f^2)}{f_n^4 + f^4} \tag{3-13}$$

Here f_n is called the corner frequency of the loop and Equation (3-12) represents a second-order loop. Higher-order PLLs may also be considered, i.e., $|1 - H(f)|^2 = f^{2k}/(f_n^{2k} + f^{2k})$. Such loops are **sometimes** plagued by stability problems and since a second-order loop can handle f^{-3} phase noise it represents a good system choice. Equations (3-11) and (3-12) are plotted in Figure (3-2).

The loop bandwidth B_φ is directly proportional to the corner frequency f_n. It is defined as the equivalent noise bandwidth of H(f), that is,

$$B_\varphi = \int_0^\infty |H(f)|^2\, df$$

$$= f_n \int_0^\infty \frac{1 + 2v^2}{1 + v^4}\, dv \qquad (v = \frac{f}{f_n})$$

$$= f_n \cdot \frac{3\pi\sqrt{2}}{4} = f_n \cdot 3.33 \tag{3-14}$$

Since f_n is directly proportional to the loop bandwidth B_φ, the transfer function H(f) is indirectly specified by B_φ so the task of minimizing the total phase noise variance σ^2_{tot} reduces to one of finding the optimum loop bandwidth B_φ.

Up to this point, no consideration has been given to the fact that the phase-locked loop must be implemented to operate on a modulated signal (except for

auxiliary carrier systems). For power loops, Equation (3-10b) is unaffected but Equation (3-10c) should be multiplied by η_φ^p, the degradation factor associated with power loops.[6] Similarly, in the decision feedback implementation case, Equation (3-10c) is modified by the multiplier η_φ^d for decision-feedback loop.[8,9]*

In any case, optimization of loop bandwidth requires minimization of the total phase noise variance at the decision point. Total phase noise variance at the decision point may be written as:

$$\sigma^2_{tot} = \sum_{j=0}^{3} h_j A_j + \frac{N_o \eta_\varphi}{E_s R_s} \left[f_n^4 A_4 - 2 f_n^2 A_2 \right] \tag{3-15}$$

where:

$$A_j = \int_0^\infty \frac{f^{4-j}}{f^4 + f_n^4} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df \tag{3-16}$$

and $j = 0, 1, 2, 3, 4$

Evaluation of integrals A_j is simplified by letting $\alpha = \pi f_n T_s$ and $v = f/f_n$ giving:

$$I_{4-j} = f_n^{j-1} A_j = \int_0^\infty \frac{v^{4-j}}{1+v^4} \left(\frac{\sin \alpha v}{\alpha v} \right)^2 dv \tag{3-17}$$

Integrals I_k ($k = 4-j$) have been evaluated in Appendix A taking into account that the greatest interest is for small α-values. Results are tabulated below.

$$A_0 f_n^{-1} = I_4 = \frac{\pi}{2} \left[\frac{1}{\alpha} - \frac{1}{\sqrt{2}} \cdot \frac{1}{3\sqrt{2}} \alpha^2 \cdot \ldots \right]$$

$$A_1 = I_3 = \frac{1}{\alpha} \cdot 0.24 \cdot 1.04 \alpha^2 \cdot \ldots$$

$$f_n A_2 = I_2 = \frac{\pi\sqrt{2}}{4} \left[1 - \frac{2\sqrt{2}}{3} \alpha \cdot \ldots \right]$$

*See also Appendix A.

$$f_n^2 A_3 \approx I_1 = 0.83 + \frac{2}{3}\alpha^2 \ln\alpha \cdot \ldots$$

$$f_n^3 A_4 \approx I_0 = \frac{\pi\sqrt{2}}{4}\left|1 - \frac{1}{3}\alpha^2 \cdot \ldots\right| \qquad (3\text{-}18)$$

For small α values substitution of Equations (3-18) into (3-15) gives the total phase noise variance at the symbol decision point

$$\sigma^2_{tot} \approx h_0 \frac{\pi f_n}{2\alpha} + h_1 (0.24 + \ln\frac{1}{\alpha}) + h_2 \frac{\pi\sqrt{2}}{4 f_n} + h_3 \frac{0.83}{f_n^2}$$

$$+ \frac{N_0 \eta_\varphi}{E_s R_s} \cdot f_n \frac{3\pi\sqrt{2}}{4} \qquad (3\text{-}19)$$

Since $B_\varphi = f_n \frac{3\pi\sqrt{2}}{4}$ and $\alpha = \pi f_n T_s = \pi f_n R_s^{-1} = \frac{4}{3\sqrt{2}} \frac{B_\varphi}{R_s}$, we may rewrite Equation (3-19) as

$$\sigma^2_{tot} \approx h_0 \frac{R_s}{2} + h_1 \left[0.3 + \ln\frac{R_s}{B_\varphi}\right]$$

$$+ h_2 \frac{3.70}{B_\varphi} + h_3 \frac{9.22}{B_\varphi^2} + \frac{N_0 B_\varphi}{E_s R_s} \eta_\varphi \qquad (3\text{-}20)$$

The optimum bandwidth B_φ that minimizes the total variance σ^2_{tot} can be found from this expression. Setting the derivative of σ^2_{tot} with respect to B_φ equal to zero the optimum value of

$$B_\varphi = B_0 + \frac{1}{3} h_1 \frac{E_s R_s}{N_0 \eta_\varphi} \qquad (3\text{-}21)$$

B_φ represents the optimum bandwidth, where

$$B_0 = \sqrt[3]{\frac{1}{2}\left(G + \sqrt{G^2 - 4H^3}\right)} \qquad (3\text{-}22a)$$

if real and

$$B_0 \simeq 2\sqrt{H}\ \cos\left[\frac{1}{3}\ \arccos\left(\frac{G}{2\sqrt{H^3}}\right)\right] \tag{3-22b}$$

otherwise [10]. Here

$$H = \frac{1}{3}\left(\bar{h}_2 + \frac{1}{3}\,\bar{h}_1^2\right) \tag{3-23}$$

and

$$G = 2\left[\bar{h}_3 + \frac{1}{6}\ \bar{h}_2\,\bar{h}_1 + \frac{1}{27}\ \bar{h}_1^3\right] \tag{3-24}$$

with

$$\bar{h}_1 = h_1\,\frac{E_s R_s}{N_o \eta_\omega} \tag{3-25}$$

$$\bar{h}_2 = h_2\ 3.7\,\frac{E_s R_s}{N_o \eta_\omega} \tag{3-26}$$

$$\bar{h}_3 = h_3\ 9.22\,\frac{E_s R_s}{N_o \eta_\omega} \tag{3-27}$$

Generally, the optimum bandwidth solution is approximated by

$$B_\omega = \sqrt[3]{2\,\bar{h}_3} + \frac{1}{2}\,\bar{h}_1 \tag{3-28}$$

for the very low symbol rates R_s. By increasing symbol rates the optimum solution of Equation (3-21) is closely approximated by

$$B_\omega \quad \frac{1}{2}\ \bar{h}_1 + \sqrt{\bar{h}_2 + \bar{h}_1^{\,2}/4} \tag{3-29}$$

that finally will approach

$$B_\omega = \bar{h}_1 \tag{3-30}$$

This general trend implies that systems operating at high digital rates will not be plagued by f^{-3} phase noise because it is effectively tracked out by the phase-locked loop. Having determined the optimum loop bandwidth, one can calculate the resulting total phase noise variance according to Equation (3-20).

Before applying this optimization technique to available phase noise data, it is of great value to present the method by which the optimum choice of the loop bandwidth distributes the total phase noise variance between that due to phase noise on the received signal and the thermally induced loop phase noise. For this purpose we consider a simplified model of the total phase noise variance given by

$$\sigma^2 = \frac{H}{(s-1)\, x^{s-1}} + Lx \qquad (s > 1) \qquad \{R_s >> x\} \qquad (3\text{-}31)$$

where

$$h_s = \frac{H}{s-1}$$

and

$$\sigma^2 = h_1 \left[.3 - \ln \frac{R_s}{x} \right] + Lx \qquad (s \quad 1) \qquad (3\text{-}32)$$

and where x is proportional to the noise bandwidth of the loop (see Equation (3-20)). In other words, we are optimizing the bandwidth of a phase-locked loop in the presence of f^{-s} noise (the $\frac{H}{(s-1)\, x^{s-1}}$ term or the $h_1 \ln \frac{R_s}{x}$ term) and additive noise (the Lx term). The value x_s of x that minimizes σ^2 is the solution of

$$\frac{\delta \sigma^2}{\delta x} = - \frac{H}{x^s} + L \quad 0 \qquad (s > 1) \qquad (3\text{-}33)$$

or

$$\frac{\delta\sigma^2}{\delta x} = -\frac{h_1}{x} + L = 0 \qquad (s = 1) \tag{3-34}$$

yielding

$$x_s = \sqrt[s]{H/L} \qquad (s > 1) \tag{3-35}$$

and

$$x_1 = h_1/L \qquad (s = 1) \tag{3-36}$$

For $(s > 1)$ the optimum solution distributes the total phase variance (at the symbol decision point) between the untracked portion of carrier phase noise and that due to thermal noise in the proportion $(1/s)$ to $(s-1)/s$ giving the minimum total variance

$$\sigma_s^2 = \left(\frac{s}{s-1}\right) L x_s = \frac{s}{s-1} \sqrt[s]{HL^{s-1}} \qquad (s > 1) \tag{3-37}$$

For $(s = 1)$ the optimum solution is slightly more complicated due to the very slow roll off in power for a $1/f$ type phase noise density. The minimum total phase noise variance is obtained using Equation (3-32).

$$\sigma_1^2 = h_1 \left[.3 + \ln \frac{R_s}{x_1}\right] + L x_1 \qquad \left\{\frac{R_s}{x_1} \gg 1\right\} \tag{3-38}$$

and using Equation (3-36) gives

$$\sigma_1^2 = h_1 \left\{\left[.3 + \ln \frac{R_s}{x_1}\right] + 1\right\} = h_1 \ln 3.6 \frac{R_s}{x_1} \qquad \left\{\frac{R_s}{x_1} \gg 1\right\} \tag{3-39}$$

For any typical coherent PSK system it is reasonable to assume that the ratio of modulation symbol rate to PLL bandwidth (R_s/x_1) will be at least > 5 and is typically > 10.* Assuming $R_s/x_1 > 5$ or equivalently, $\ln(R_s/x_1) > 1.6$

* If the optimum bandwidth $x_1 \approx R_s$ there is no reason to design a "coherent" PSK system since there would be no advantages accrued over performance obtainable from a differentially coherent PSK system which would be much simpler to implement.

and using Equation (3-39) the optimum solution distributes the total phase noise variance between the untracked portion of carrier phase noise and that due to thermal noise in the proportion γ to $(1-\gamma)$ respectively where $(.65 < \gamma < 1)$.

In the preceding analysis, the optimum tradeoff between colored phase noise $h_i/f^i{}_{i\ 1,..3}$ and thermal noise has been determined. As discussed in Section 2 of this report, the tradeoff between allowable white oscillator phase noise and additive thermal noise is perhaps the most misunderstood pro- process. It is hoped that the following simple discussion will clarify any conceptual difficulty.

Equation (3-20) shows when the oscillator phase noise process is dominated by white phase noise (i.e., $h_0 \ggg h_i$, i = 1, 2, 3) the total phase error variance becomes:

$$\sigma^2_{tot} \simeq h_0 \frac{R_s}{2} + \frac{N_o B_\varphi}{E_s R_s} \eta_\varphi \qquad (3-40)$$

However, the first term on the left is an approximation of the filtered white phase noise process which only holds when the PLL corner frequency f_n is much less than the symbol rate R_s. When this condition does not hold Equation (3-40) should be given as:

$$\sigma^2_{tot} \simeq h_0 \left(\frac{R_s}{2} - f_n\right) + \frac{N_o B_\varphi}{E_s R_s} \eta_\varphi \qquad (3-41)$$

where it is assumed that

$$R_s/2 > f_n$$

and that $(R_s/2 - f_n)$ is the 3-dB bandwidth of the composite phase noise filter consisting of the PLL filter and integrate-and-dump filter.

A very interesting result is obtained by a simple rearrangement. Noting that $B_\varphi \approx 3.33\ f_n$ (second order PLL, $\zeta \simeq .707$)

$$\sigma^2_{tot} = h_0 \frac{R_s}{2} + B_\omega \left(\frac{N_0}{E_s R_s} \eta_\omega - \frac{h_0}{3.33} \right) \qquad (3-42)$$

From Equation (3-42) it is seen that by a judicious choice of parameters it is possible to cause the bracketed quantity to be positive, negative, or zero thus indicating that by increasing the PLL bandwidth B_ω with a fixed symbol rate R_s, it is possible to cause the carrier reference total phase error variance to increase, decrease, or remain the same, respectively. Of course this only holds when $f_n < R_s/2$ which is a usual requirement for coherent PSK demodulation.

If a set of parameters given are such that the bracket quantity is negative, and the optimum PLL bandwidth thus approaches the symbol rate (noise averaged over only one symbol duration) it is obvious that coherent PSK demodulation holds no advantage over differentially detected PSK since all of the noise of the previous bit interval will appear on the phase reference. Should this situation occur it would be wise to switch to a differentially detected PSK modem and remove the differential decoders normally used to resolve the phase ambiguity problem in coherent PSK systems.

The preceding results are summarized in Table 3-1 which gives the relative phase noise distribution for various s-values that occur. The results of this optimization also indicate, for example, in the case of dominating f^{-3} phase noise, the loop bandwidth should be set so large that only one-third of the total phase noise originates from the carrier phase noise. In addition, even though only one phase noise characteristic was considered here (f^{-s} phase noise), it is clear that from a bandwidth optimization point of view the important consideration is the characteristic of the phase noise about the corner frequency f_n of the loop (see Equation (3-19)). From a purely analytical point of view, it can be argued that the only things that matter to achieve the MMSE is the differential gain or loss of the total phase noise variance obtained by varying the loop bandwidth.

Table 3-1. Optimum Distribution of Carrier Reference Phase Error Variance

Phase Noise Characteristic	Portion of Phase Noise Variance Due to Carrier Phase Noise		Portion of Phase Noise Variance Due to Thermal Noise		Assumptions
f^0	1	0 dB	0	$-\infty$ dB	$h_0 > 3.33 \frac{N_o}{E_s R_s} \eta_\omega$
	0	$-\infty$ dB	1	0 dB	$h_0 < 3.33 \frac{N_o}{E_s R_s} \eta_\omega$
f^{-1}	0.65 to 1.0	-2.1 to 0 dB	0.35 to 0	-1.6 dB to $-\infty$ dB	$R_s/B_\omega > 5$
f^{-2}	1/2	-3 dB	1/2	-3 dB	
f^{-3}	1/3	-5 dB	2/3	-2 dB	
f^{-4}	1/4	-6 dB	3/4	-1.3 dB	

3.2 ANALYTICAL DIFFICULTIES AND THEIR RESOLUTION

The analytical optimization procedures described in the preceding section assume well behaved spectral density shapes with monotonically non-increasing density versus frequency and, also assume integer values for the exponent which describes the slope of the phase noise curve. It becomes apparent, however, from measured performance data that various filtering techniques used in real equipment do not always provide such well behaved phase noise spectra. Two techniques are available to supplant the preceding analysis when necessary. One technique which is currently available [11] is a graphical solution to the equations of the preceding section (computer integration) and the second technique is a state variable solution to the phase noise problem.[12] The state variable solution also provides the capability for studying the effects of time gated operation, required for TDMA systems. However, computer graphical procedures are perhaps the most straightforward and can be accurate for TDMA systems over a specified range of parameters as discussed in Paragraph 5.3 of this report.

In any event, the solutions derived in the preceding section provide great insight into most of the systematic variations experienced. In the following sections, any one of the preceding analytical tools is used depending upon which is judged best for the particular application.

SECTION 4 - OPTIMUM PERFORMANCE OF BPSK AND QPSK SIGNALING WITH VITERBI (RATE 1/2, K = 7) DECODING IN THE PRESENCE OF OSCILLATOR PHASE NOISE EXPECTED FOR TERMINALS OF THE DSCS

4.1 OPTIMUM PERFORMANCE RESULTS WITH TERMINALS OBEYING THE "HT-MT" MODIFIED PHASE NOISE SPECIFICATION

Since the advent of practical coherent PSK modulation techniques for the DSCS has been relatively recent, the "original" phase noise specifications for the HT-MT earth terminals of the DSCS were derived from an incidental FM model as stated below in a paragraph from SCA-2080A.

> Incidental FM. Transmitted carrier and receive carriers after frequency translation shall be spectrally pure so that:
>
> $\Delta f \times f_m$ is not greater than 2 Hz squared for values of f_m between 1.0 Hz and 20 Hz. For values of f_m above 20 Hz, Δf shall not exceed 0.1 Hz to the value of f_m where the single-sided phase noise density to signal ratio equals -105 dB. The -105 dB single-sided phase noise density to signal ratio shall not be exceeded from 10 kHz to 62.5 MHz on either side of carrier Δf = peak deviation of the carrier f_m = deviation rate.

Using this model the HT-MT single sideband phase noise $\mathcal{L}(f)$ was derived [13] and is shown in Figure 4-1. It is known, however, that the f^{-4} phase noise indicated in close to the carrier is unrealistic given currently available oscillators and that this type of spectral shape is due to the assumed validity of the incidental FM model. Frequency synthesizers are known to provide an f^{-1} density close to the carrier and are ultimately limited by the effects of either crystal or atomic standards which exhibit f^{-3} phase noise characteristics extremely close to the carrier. Thus, the HT-MT specification has been modified for the purposes of this memorandum as shown by the broken line curve of Figure 4-1 labelled HT-MT-modulation.

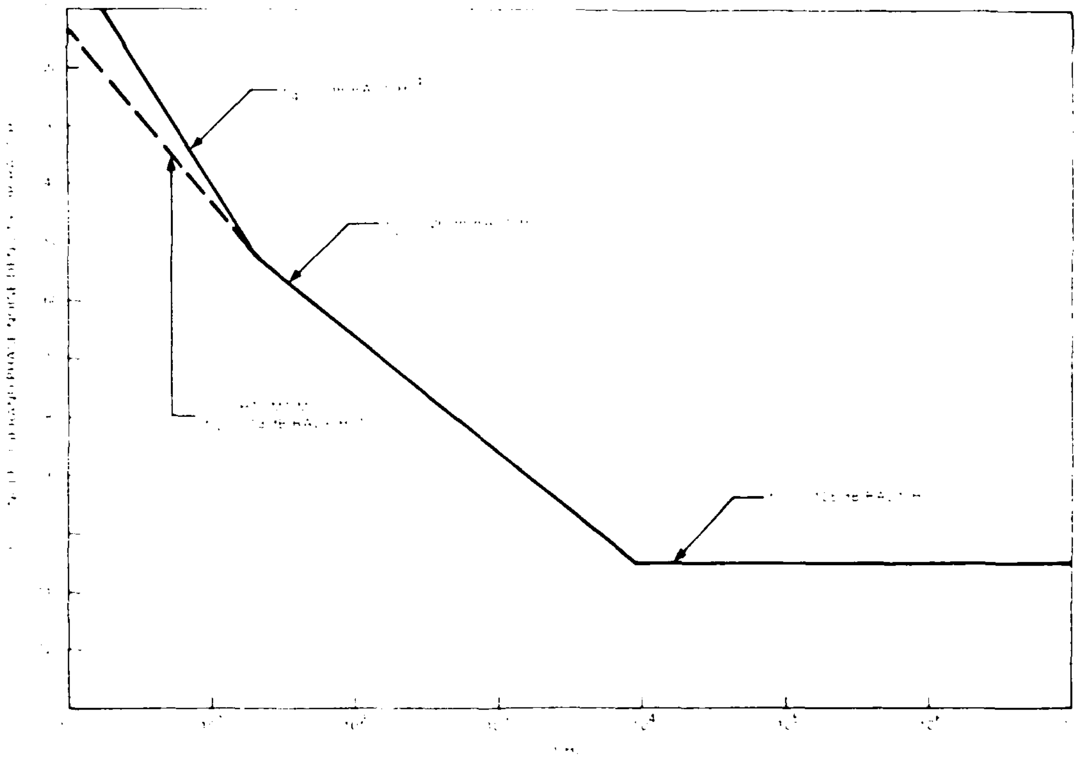

Figure 4-1. HF-MF Incidental FM Specification Converted to Single Sideband Phase Noise Density ℒ(f) at Carrier Frequency

Based on this modified specification, the total phase noise variance given by Equation (3-20) has been determined using the optimum bandwidth solution of Equation (3-21). The variance has been calculated using both decision-feedback and power loop implementations for two and three times the terminal phase noise according to the specification, since two terminals and one satellite are always involved. This calculation is used because reliable phase noise data about the satellite is not available. (Some very sketchy and incomplete information pertaining to the satellite phase noise is available, but by neglecting the satellite influence and equating the satellite as one terminal, the influence of the satellite can be assessed.) Figures 4-2 through 4-5 illustrate results for two and three terminals, power PLLs, decision-feedback loops, and also for both BPSK ($M = 2$) and QPSK ($M = 4$) operations. Demodulation losses have been plotted as a function information bit rate R_{ib} for 8-level soft decision (3-bit), rate 1/2, constraint length 7, Viterbi decoding followed by differential decoding with a resultant BER 10^{-5}.

A loss cutoff of 0.2 dB has been drawn in the figures to indicate allowable regions of operation for the various configurations. The upward trend in the loss curves with high data rates are caused by the white phase noise floor shown in Figure 4-1.

Figures 4-2 through 4-5 show that when carrier recovery is provided by decision feedback loops a reduction in demodulation losses is obtained as compared with results for power loop implementations. However, significant improvement is only obtained with QPSK with negligible improvement noted with BPSK.

Table S-1 contains a summary of permissible (0.2 dB maximum loss) rates for two terminals and one equivalent satellite all conforming to the HT-MT modulation phase noise specifications (Figure 4-1).

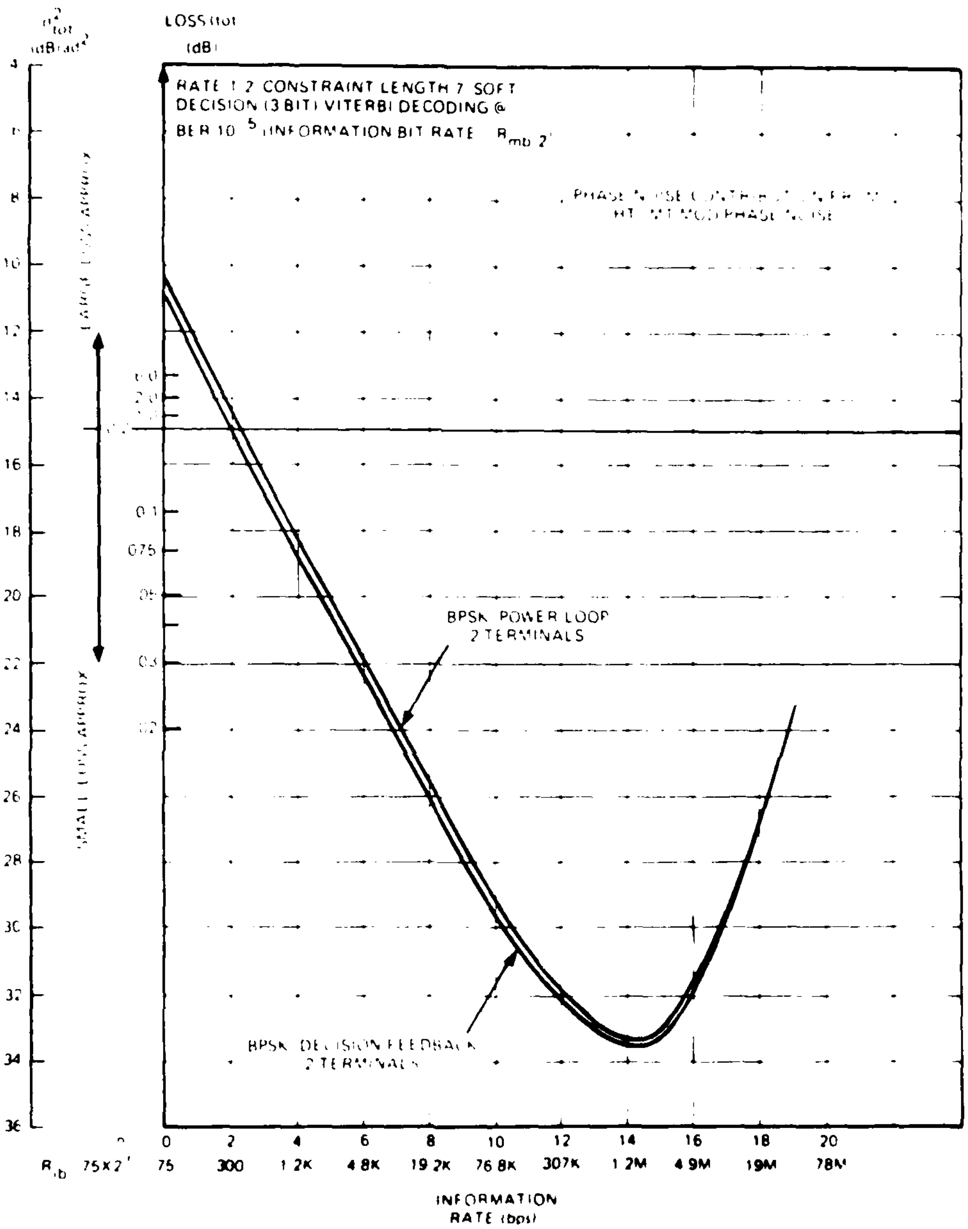

Figure 4-2. Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps), at Optimum BW

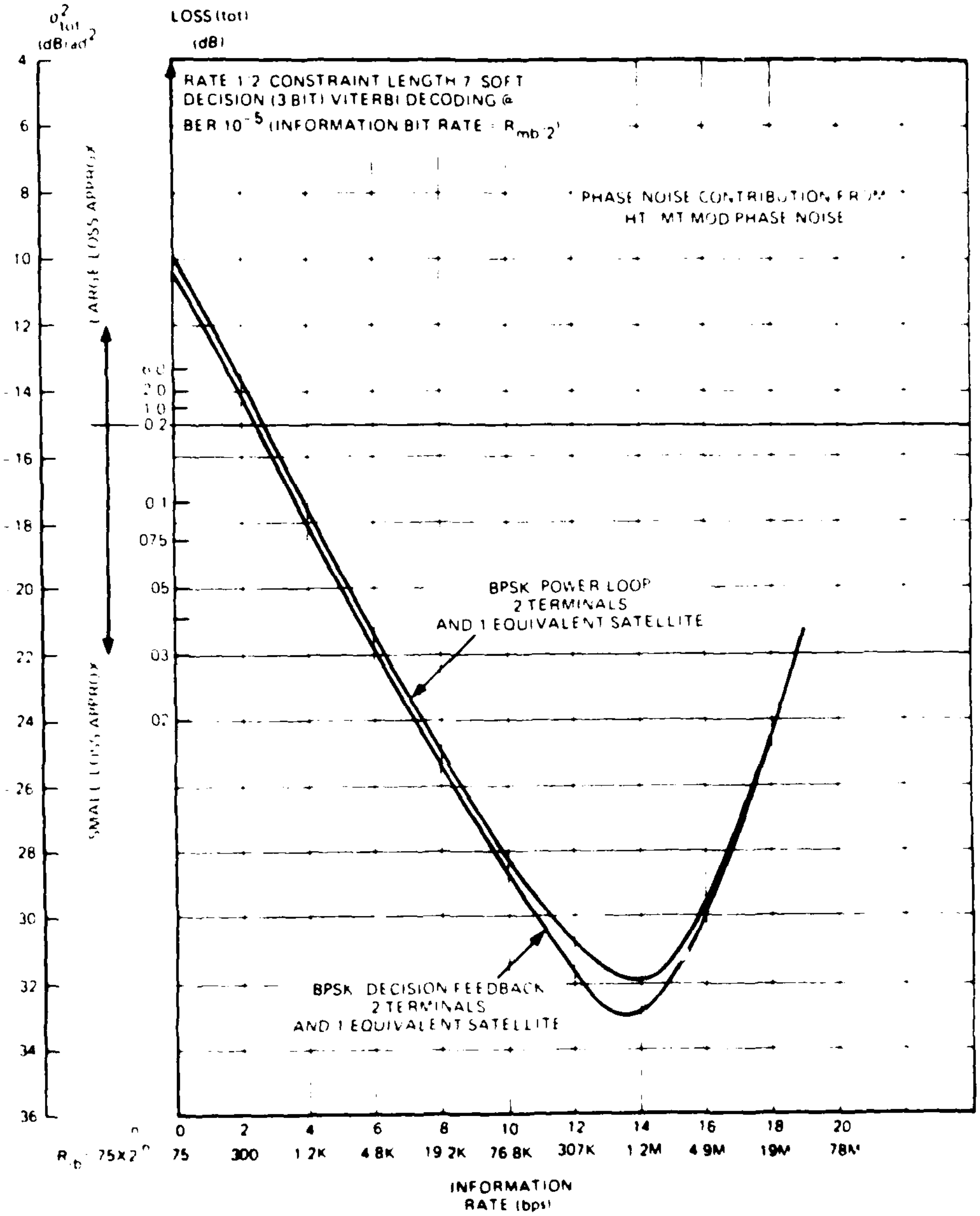

Figure 4-3. Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps), at Optimum BW

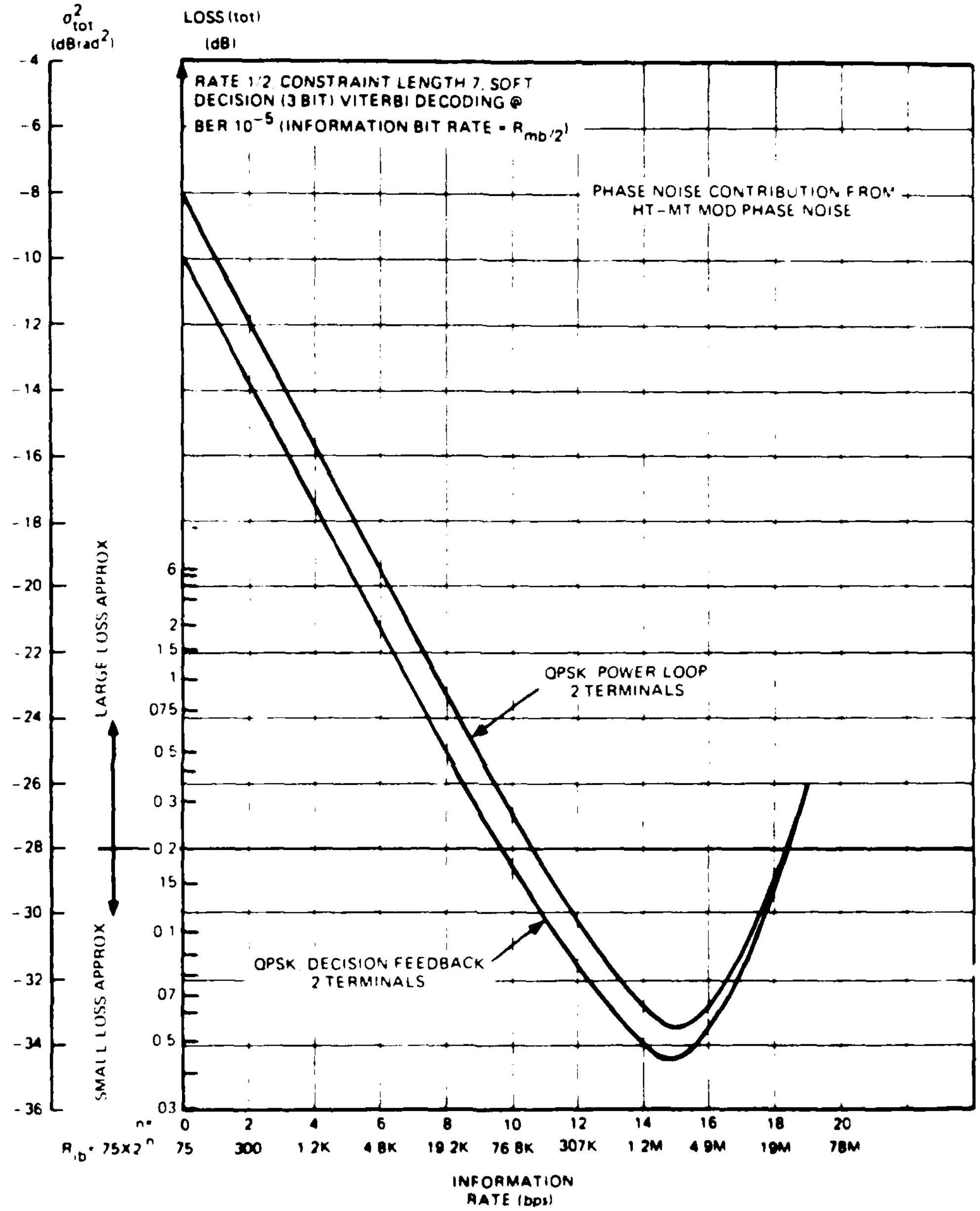

Figure 4-1. Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps), at Optimum BW

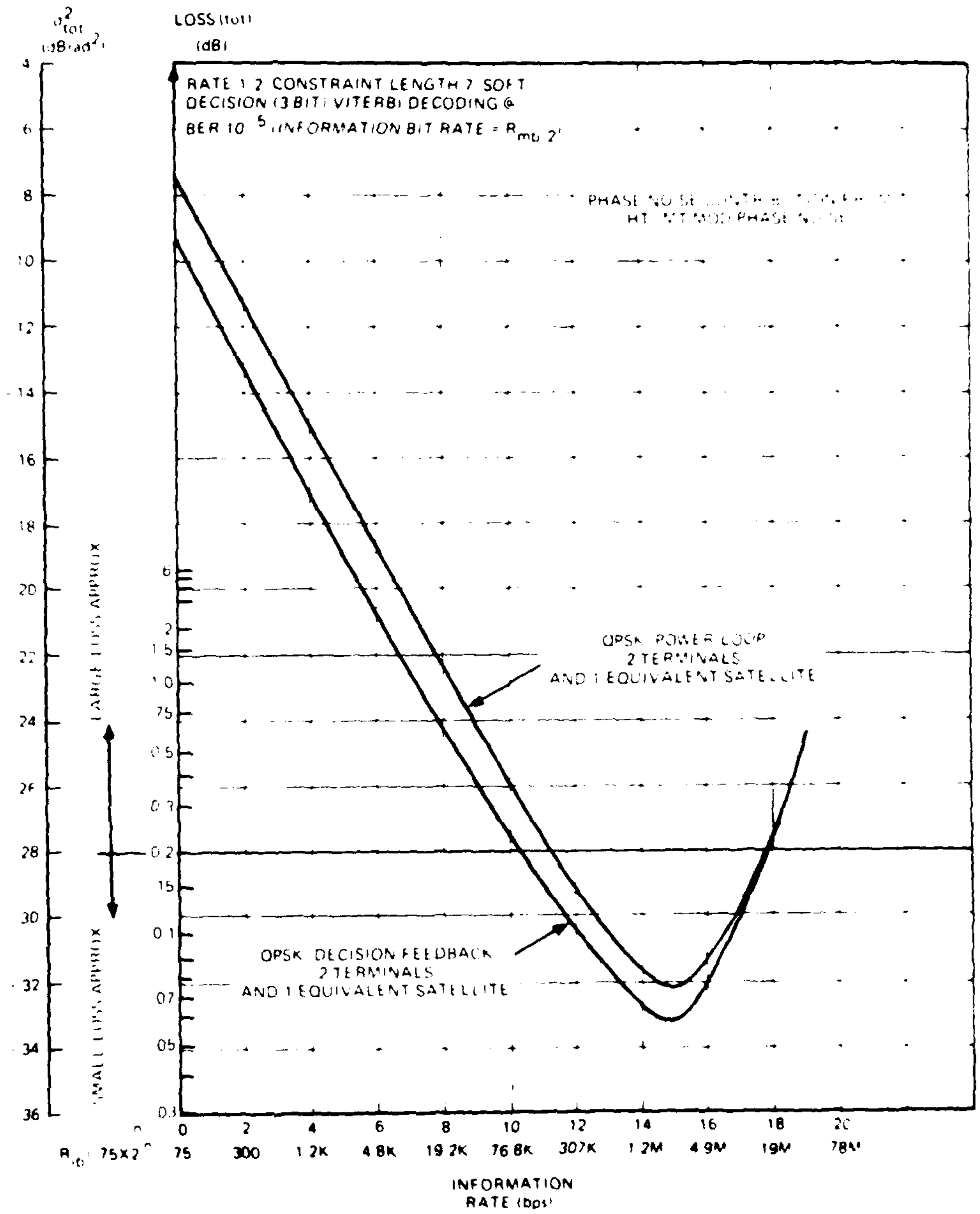

Figure 4-5. Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps) at Optimum BW

Tables D-1 through D-8 provide back-up data for the results shown in Figures 4-1 through 4-5. However, the tabulated results are expressed as a function of modulation bit rate R_{mb} as compared to information bit rate R_{ib} shown in the figures where the two parameters are related as:

$$R_{ib} = R_{mb}/2$$

In the tables the optimum bandwidth, total phase noise variance and its two components (thermally induced loop phase noise σ^2_{th} and the untracked portion of the composite oscillator phase noise spectrum σ^2_{pn}) are given as a function of the modulation bit rate.

Also tabulated are the demodulation losses that would be experienced when using 8-level soft decision (3-bit), rate 1/2, constraint length K = 7, Viterbi decoding followed by differential decoding based upon two different approximations to the probability distribution of phase tracking error experienced in a PLL.* Results based on a Gaussian approximation [8] are easily calculated but are only valid for small losses as shown in the tables while the results using a Tikhonov approximation [2] are valid when the losses are less than 6 dB.

4.2 OPTIMUM PERFORMANCE WITH REALISTIC TERMINAL PHASE NOISE

4.2.1 Phase Noise Synthesized Using Comtech Lab., Inc. L-Band Oscillator, Fluke Frequency Synthesizer and Selected Atomic and Crystal Standards

Figure 4-6 shows the general structure for deriving a 7800-MHz signal from a 5-MHz standard and Figure 4-7 shows their corresponding single sideband phase noise densities at 7800 MHz.**

*See Appendix A.

**It should be noted that for straight frequency multiplication, 10 log M^2 (dB) where M = new frequency/old frequency is added to original specifications when required.

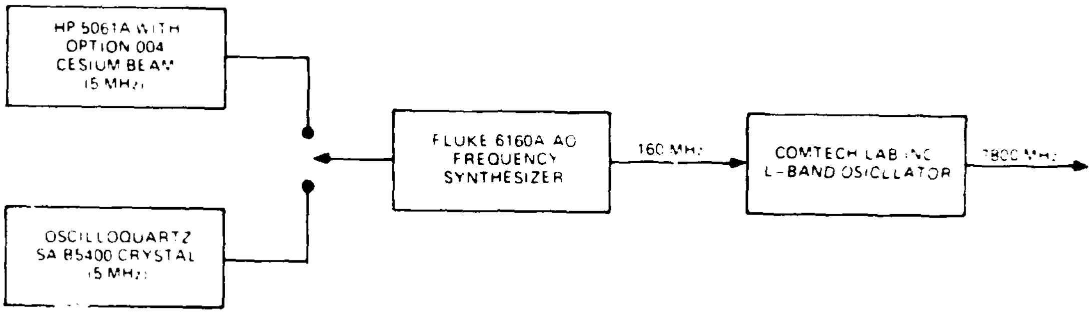

Figure 4-6. L-Band Reference Signal Generator

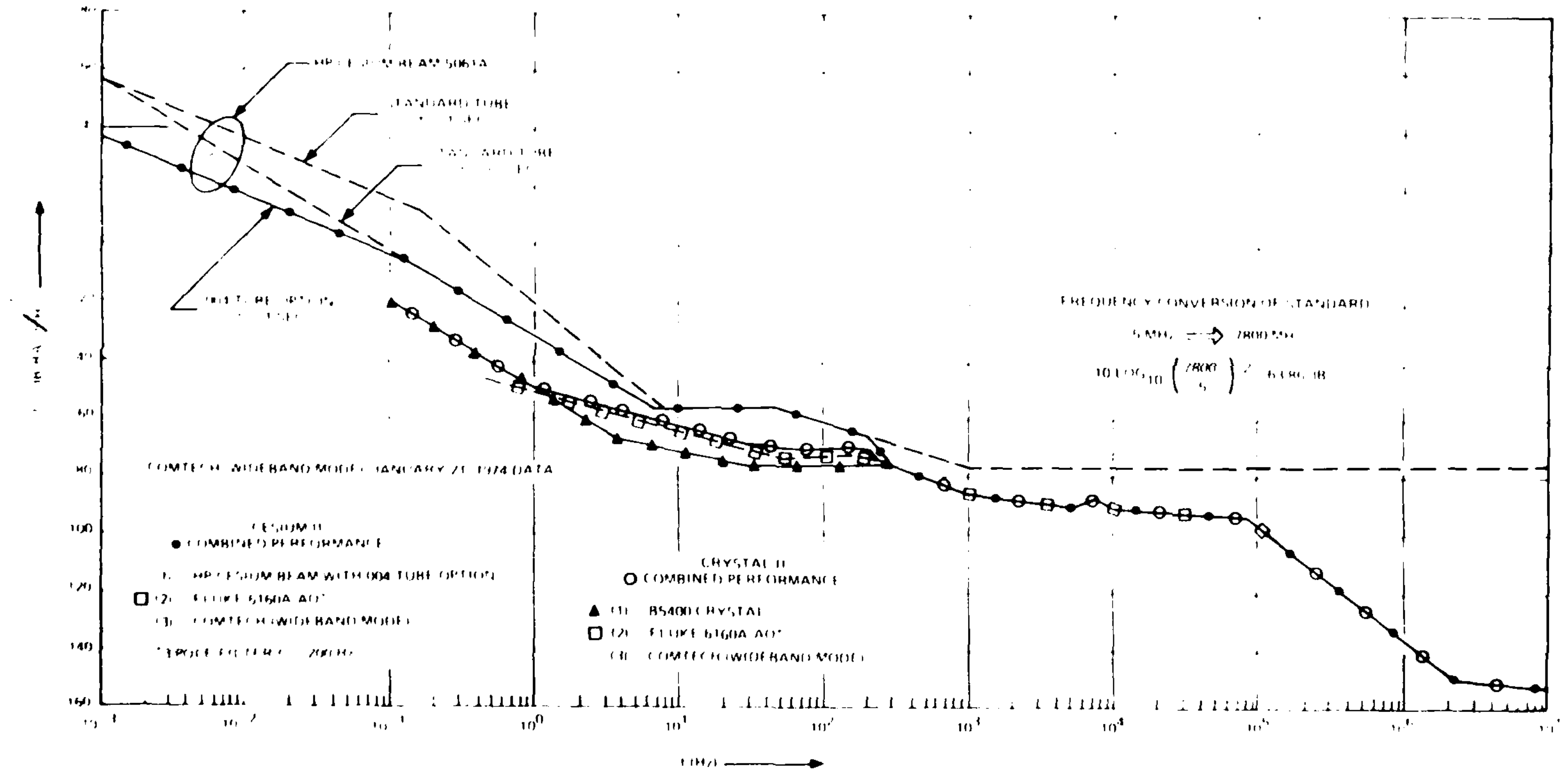

Figure 4-7. Synthesized Single Sideband Phase Noise Densities $|\mathcal{L}(f)|$ for "Cesium II" and "Crystal II" at 7800 MHz

The Hewlett Packard HP 5061 A[14] and the Oscilloquartz SA B5400[15] have been chosen as representative of state of the art portable cesium beam standards and high quality crystal references, respectively.

As noted by Hewlett Packard, 60 second time constant operation* (see Figure 4-5) requires a carefully controlled environment. Therefore, for field operations the 1 second time constant operation seems practical. Since it is expected that the optional (004) beam tube may be used in the Phase II DSCS, results obtained here will pertain only to the optional 004 beam tube with a 1 second time constant.

At this point in time it is not known whether the Oscilloquartz B5400 crystal could meet its specified performance (Figure 4-7) under field conditions, however, for the purpose of illustration it will be assumed that these conditions can be satisfied with adequate margin.

A Fluke 6160 A/AO frequency synthesizer has been chosen as representative of high quality synthesizers and provides the required flexibility with respect to frequency assignment. It is assumed that the standard drives the synthesizer and only wideband phase locking ($\gtrsim$100 kHz) is involved within the synthesizer. In accord with discussions between CSC and a Fluke representative, and as verified by Comtech, a three-pole filter with 3-dB corner frequency at 200 Hz exists within the synthesizer.

Single sideband phase noise data for the Fluke synthesizer has not been shown directly in Figure 4-7. However, it is incorporated in the measured data[16] provided by measurements of a Comtech Lab. L-band oscillator driven by a Fluke 6160A/AO synthesizer and the measured data is shown by the dotted curve of Figure 4-7 designated here as Comtech (Wideband Mode).[16] Using [16] and Fluke data, the dotted curve below frequencies ~100 kHz is dominated by synthesizer noise and above 100 kHz is dominated by L-band oscillator noise.

*This time constant refers to the bandwidth at which the internal 5-MHz crystal is locked to the cesium beam tube.

Measured data provided by Comtech Inc. is valid in the region 10 Hz - 10 MHz and since phase noise data is required beyond 10 MHz it has been assumed that as a worst case a phase noise floor exists at -153 dB rad^2/Hz.

In summary, two synthesized phase noise curves (labelled "cesium II" and "crystal II" in Figure 4-7) will be evaluated for generation of 7800 MHz frequency up conversion or down conversion chains. The "cesium II" curve corresponds to a frequency conversion chain driven by the HP 5061 A with optional (004) cesium beam and with 1 second time constant. The resultant single sideband phase noise is shown in Figure 4-7 and consists of three sections with frequencies below 300 Hz dominated by the atomic standard frequencies from 300 Hz-10 MHz dominated by the synthesizer L-band oscillator combination and above 10 MHz given by the assumed phase noise floor.

The second phase noise curve (designated "crystal II" in Figure 4-7) corresponds to frequency conversion driven by the Oscilloquartz SA B5400 crystal as shown in Figure 4-6. The resultant single sideband "crystal II" phase noise curve consists of four sections with frequencies below 2 Hz dominated by the crystal standard, frequencies between 2 Hz-300 Hz being a composite of noise from Fluke 6160A, AO synthesizer and crystal standard and frequencies above 300 Hz are as described for the "cesium II" curve.*

4.2.2 BPSK and QPSK System Performance Optimization With Synthesized Phase Noise Data

To our knowledge as of March 1974, single sideband phase noise curves "cesium II" and "crystal II" generated in Paragraph 4.2.1 represent the most current estimates of terminal phase noise at 7800 MHz which is expected for the best terminals currently under consideration for operation in the Phase II DSCS. Therefore an extensive set of data (Figures 4-8 through 4-11 and Tables

*A roman numeral II has been used here to differentiate this most recent data from that which appeared in a prior memorandum.

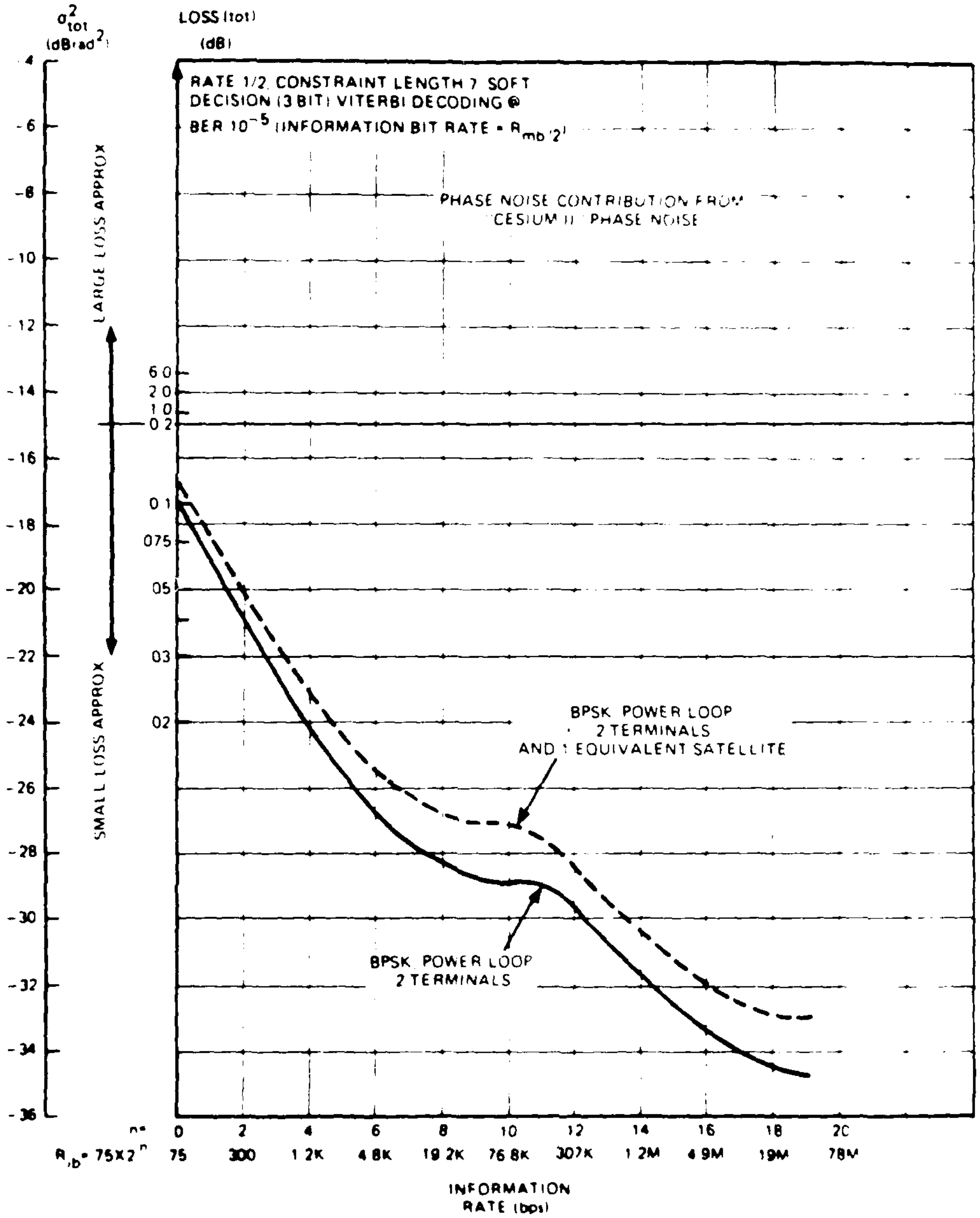

Figure 4-8. Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps), at Optimum BW

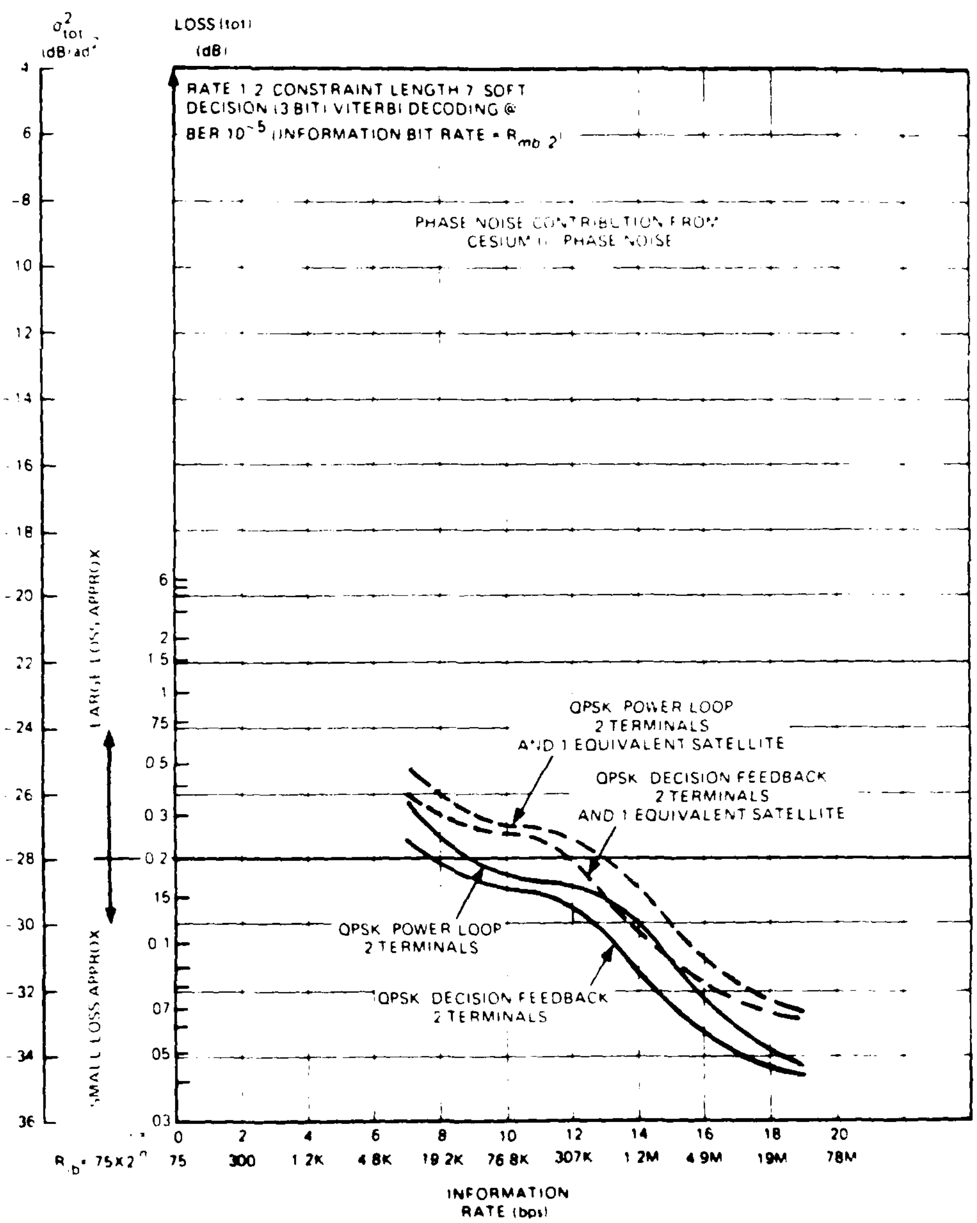

Figure 4-9. Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps) at Optimum BW

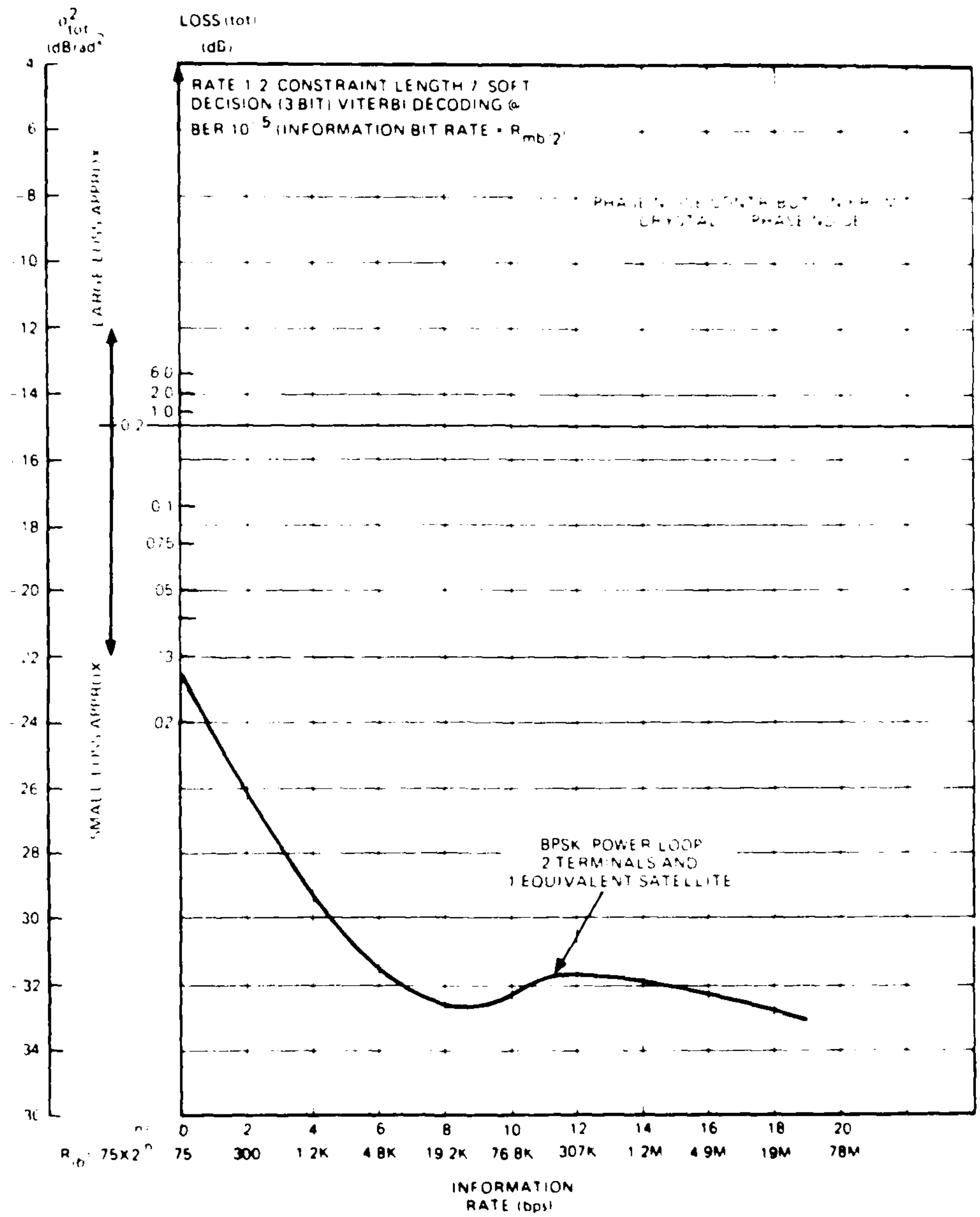

Figure 4-10. Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps), at Optimum BW

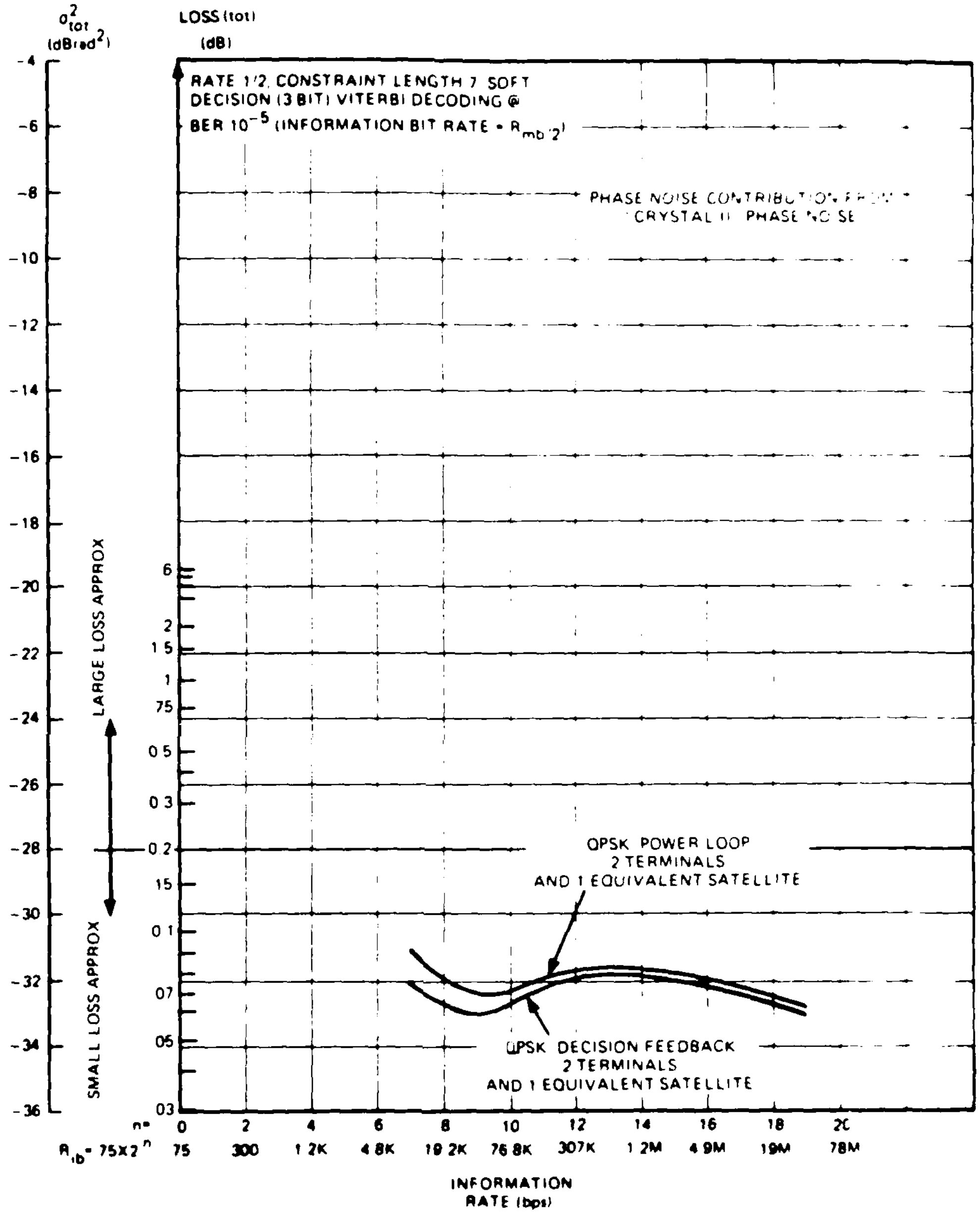

Figure 4-11. Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps), at Optimum BW

D-9 through D-17 has been generated to indicate possible demodulation performance for the Phase II DSCS.

Figures 4-8 through 4-11 illustrate demodulation losses as a function of information bit rate R_{ib} at the optimum bandwidth for two and three times the indicated phase noise ("cesium II" or "crystal II"), respectively.

As before it should be emphasized that the tabulated losses are a function of modulation bit rate R_{mb} while in the figures the losses are plotted as a function of information bit rate R_{ib} where $R_{ib} = R_{mb}/2$ due to the rate 1/2 coding procedure.

Results using "crystal II" phase noise are presented only for the case of phase noise contributed by two terminals and one equivalent satellite since these results indicate that adequate performance (< 0.2 dB loss) is achieved with all configurations (see Figure 4-10 and 4-11) over the required range of data rates.

It should be noted that results in Figures 4-8 through 4-11 do not indicate a maximum information rate limitation within the indicated range as experienced for results obtained in Paragraph 4.1 (see Figures 4-4 and 4-5). This result is due to the reduced phase noise floor of -153 dB rad^2/Hz (Figure 4-7) for "cesium II" and "crystal II" phase noise as compared to a phase noise floor of -105 dB rad^2/Hz (Figure 4-1) assumed for the HT-MT modulation phase noise. Of course, if the -153 dB rad^2/Hz floor persisted to higher frequencies and if signaling were required at higher data rates the same upward loss trend at high rates would be repeated.

Minimum and maximum data rates possible for the three terminal phase noise contribution either "cesium II" or "crystal II" have been summarized in Table S-1.

Finally, since the data in Figures 4-8 through 4-11 and Table S-1 represent demodulation losses for phase noise contributed by two terminals (no

satellite contribution) and two terminals plus one equivalent satellite, the data provides upper and lower bounds on expected losses. However, it is desirable to measure actual satellite phase noise to provide a more exact picture of expected demodulation performance losses in the Phase II DSCS.

SECTION 5 - DEMODULATION PERFORMANCE OF CURRENT MODULATION SYSTEMS OPERATING IN THE PRESENCE OF PHASE NOISE IN THE PHASE II DSCS

5.1 RADIATION INC. BPSK (MD-921 G)

Radiation Inc. has recently designed a BPSK modem which is expected to be operated with the following earth terminals of the Phase II DSCS:

1. MSC-46 "Upgrade"
2. HT-MT "Follow-on"
3. TSC-54.

Since each of the above earth terminals are expected to be operated with Comtech Lab. up- and down-converters or terminals meeting the HT-MT phase noise specifications, phase noise associated with each terminal may be adequately described by the curves labeled modified HT-MT of Figure 4-1 and "cesium II" or "crystal II" of Figure 4-7.

The Radiation BPSK modem has been designed with a power type carrier recovery PLL with a fixed bandwidth B_o = 175 Hz and damping factor ζ = 1.0.

Figures 5-1 through 5-3 have been generated to indicate expected performance of this modem operating in conjunction with soft* Viterbi decoding in the presence of the modified HT-MT Type "cesium II" and "crystal II" oscillator phase noise densities. Tables D-18 through D-23 contain the numerical support for these figures.

As in the preceding sections, data rates listed in the tables are expressed as modulation bit rate R_{mb} while the data rates shown in the figures are information bit rate where $R_{ib} = R_{mb}/2$ (see Figure 2-1).

* See note 1 of Annex.

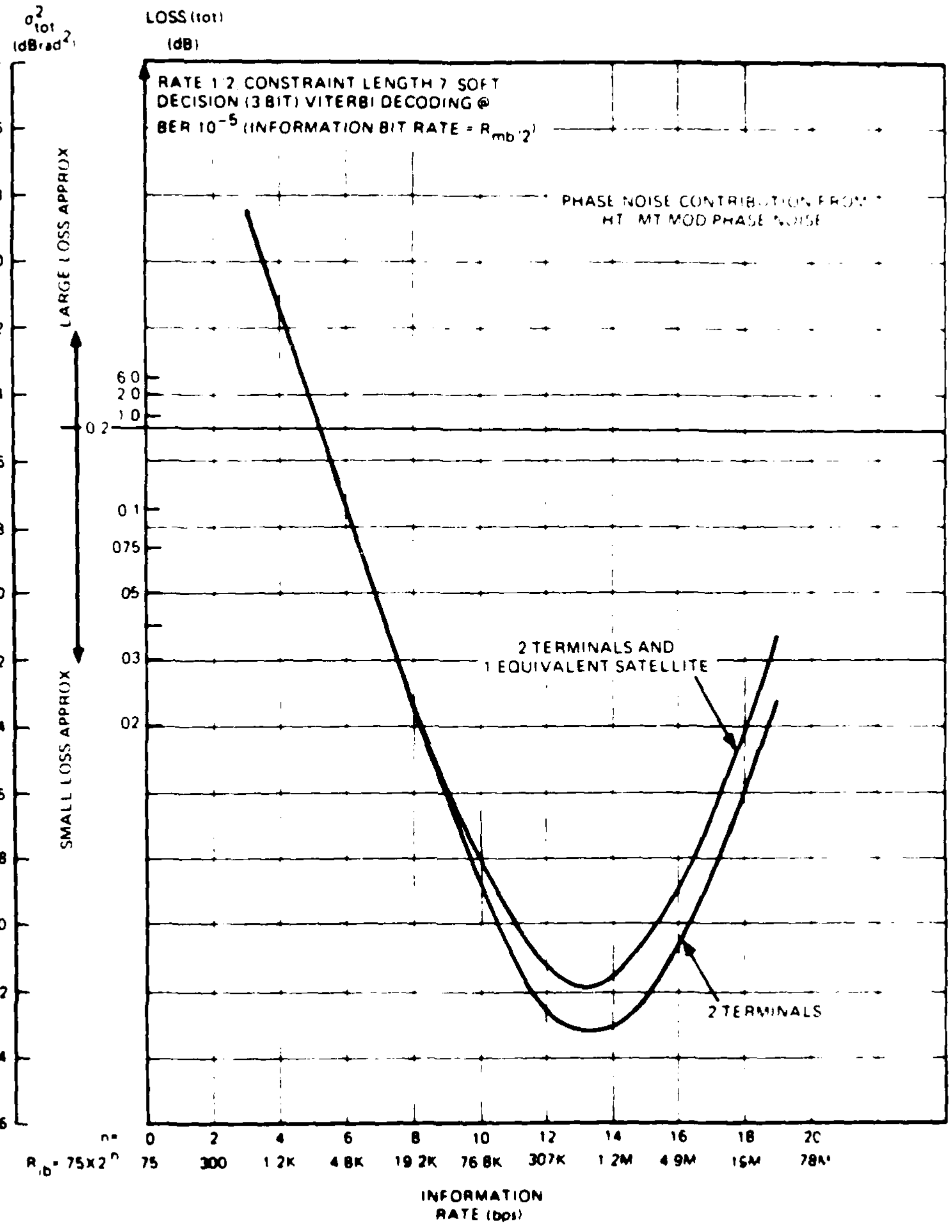

Figure 5-1. Radiation Inc. MD-921G BPSK Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps), B_φ = 175 Hz

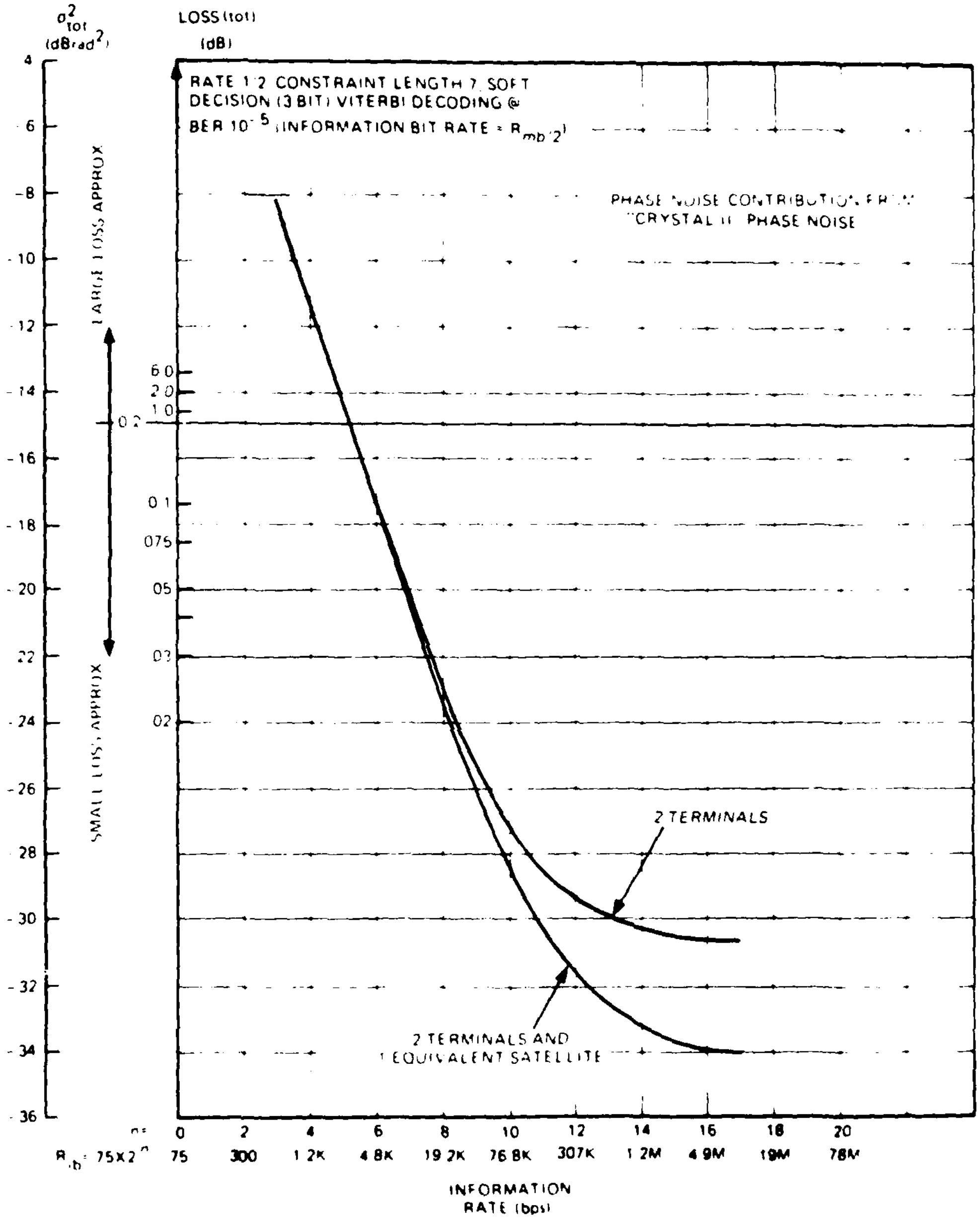

Figure 5-2. Radiation Inc. MD-921G BPSK Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib}(bps), B_φ = 175 Hz

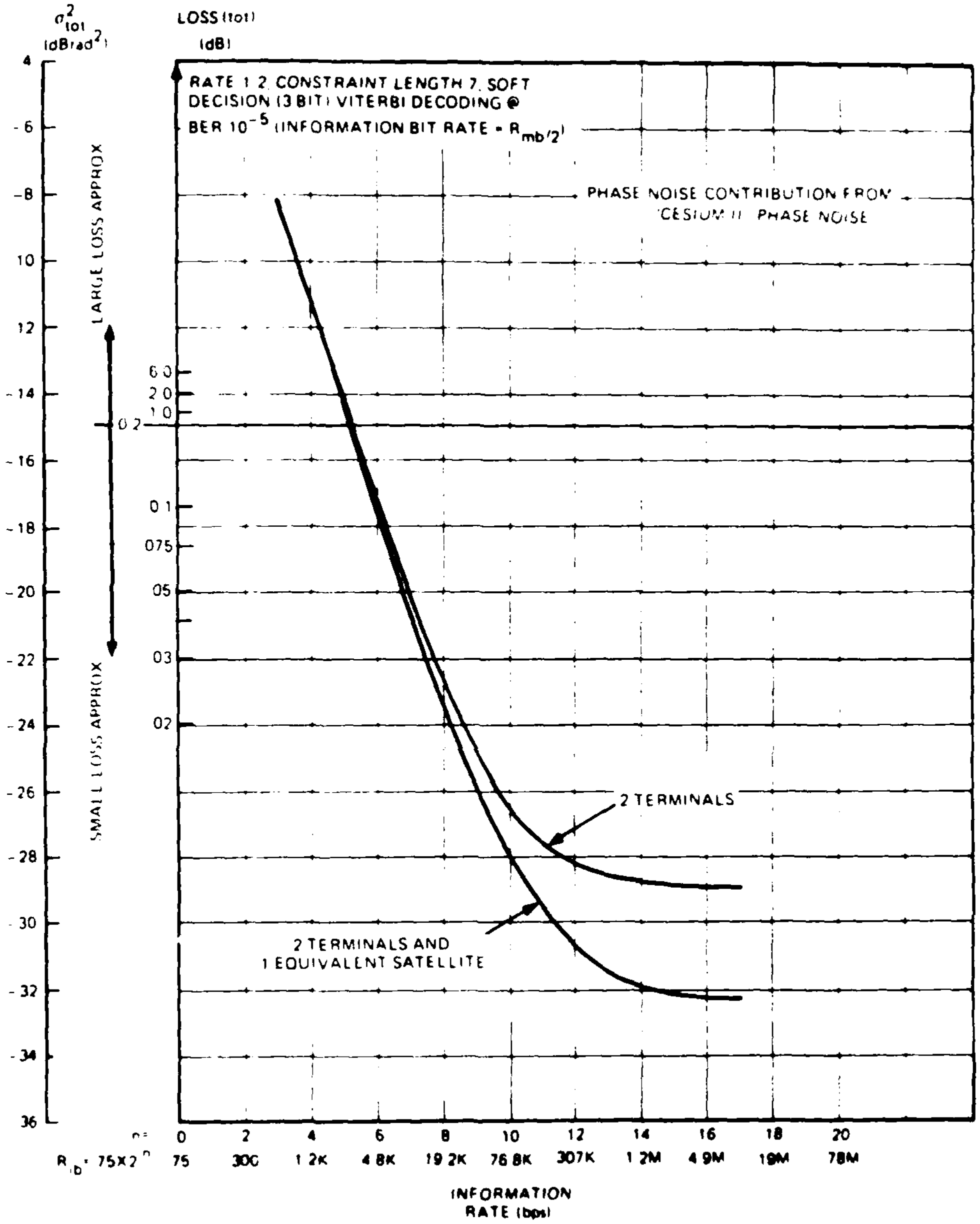

Figure 5-3. Radiation Inc. MD-921G BPSK Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps), B_φ = 175 Hz.

It may be noted in Figures 5-1 through 5-3 that a 0.707 PLL damping factor has been used to generate expected system performance data even though it has been stated that the Radiation modem has a PLL damping factor of 1.0. This change was effected here solely to reduce the costs associated with computer integration of Equation 3-10b. As stated in [11], a considerable increase in computation cost is required for PLL damping factors besides 0.707. A comparison of curves of Figures 5-2 and 5-4 (see also Tables D-22 and D-24) indicate that only a small improvement in demodulation performance is obtained with a PLL damping factor of 0.707 as compared to a PLL with damping factor of 1.0. Although differences in performance are small for loops with these two damping factors at the specified bandwidth, results are in accord with expected performance from a mean squared error criterion, that is, that optimum performance is obtained with a damping factor of 0.707.

Perusal of Figures 5-1 through 5-3 indicate that adequate demodulation performance (less than 0.2 dB loss) is achieved only when data rates are above 3600 bps.

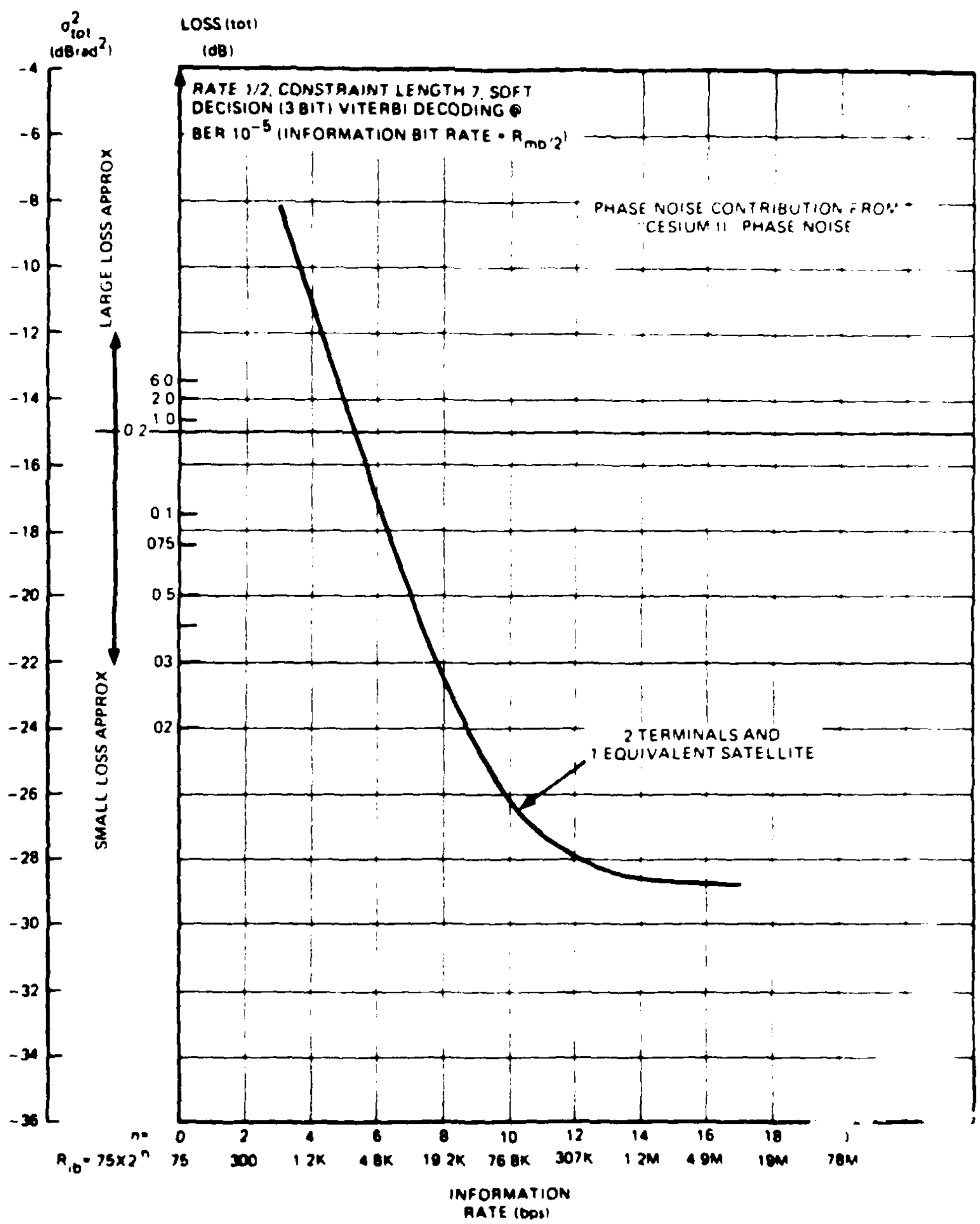

Figure 5-4. Radiation Inc. MD-921G BPSK Demodulation Loss (dB), L_{tot}, and and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps), B_φ = 175 Hz, ζ = 1.0

5.2 MAGNAVOX RESEARCH LAB., INC. (MRL) USC-28 BPSK SPREAD SPECTRUM SYSTEM

5.2.1 General

MRL's USC-28 is a BPSK spread spectrum system which consists of the following basic subsystems; Link Order Wire (LOW), Channel Data Receive Transmit (R/T), and a Critical Control Circuit (CCC).

Preceding analyses in this paper have neglected demodulation losses due to phenomenon other than imperfect carrier phase estimation. That is, losses such as those due to imperfect PSK symbol timing have been neglected. Therefore, as a continuation of this simplified analysis for the USC-28, losses due to improper PN code tracking will also be neglected and only those losses due to imperfect carrier phase estimation will be calculated.

Neglecting PN modulation, Figure 5-3 shows a simplified version of the time and power shared configuration of the LOW and R/T channels assumed in the analysis of the following sections. The CCC is a separate control circuit (not shown in Figure 5-5).

A complete analysis of the USC-28 from a phase noise point of view is provided in [1]. Our main purpose here will be to provide a simplified system analysis which will provide the basis for a USC-28 phase noise specification as discussed in the summary section and Section 6. We shall also briefly indicate expected system performance of the USC-28 operated with the HT-MT (AN/MSC-60) terminal and the MSC-46 upgrade terminal as compared to the results described in [1] for an improved version of the AN/ASC-18 terminal.

5.2.2 Phase Noise Effects in the USC-28

In Figure 5-5 it is shown that the LOW channel and R-T channel operate on a power shared basis and that carrier phase estimates are derived from the LOW and used for demodulation of data on the R-T channel. As indicated in the figure, two models of the USC-28 which are currently under discussion are the

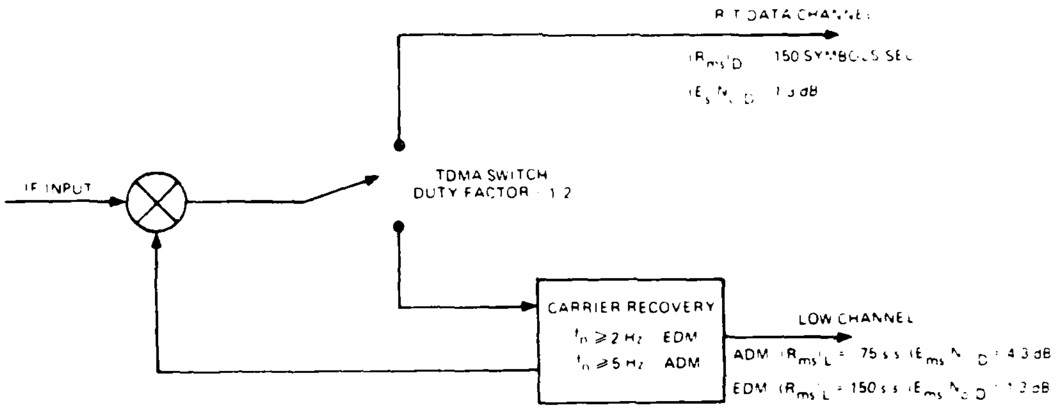

Figure 5-5. Worst Case LOW and R/T Configuration

Advanced Development Model (ADM) and the Engineering Development Model (EDM). For the purpose of our discussion, the difference between these two models is that the EDM LOW has Hamming (16, 11) coding for forward error control FEC and operates at a fixed BPSK symbol rate of 150 S/S while the ADM does not have FEC and operates at a fixed rate of 75 S/S (or equivalently 75 bps).

The R/T channel on both ADM and EDM is convolutionally encoded* and has variable data rates, the lowest being 75 bps (or equivalent 150 S/S). The TDMA duty factor switch is adjusted to provide the best power tradeoff between LOW and R/T as a function of R/T data rate. As in preceding analyses, to determine demodulation performance, we must calculate carrier phase reference quality defined as the total phase error variance σ^2_{tot}. As before, the quantity σ^2_{tot} is the sum of a phase error variance σ^2_{th} due to thermal noise and a phase error variance σ^2_{pn} due to the untracked portion of the phase noise process on the received signal. Since carrier phase estimates are obtained from the LOW, the thermally induced phase error variance is easily calculated given the LOW energy per modulation symbol/noise density $(E_{ms}/N_o)_L$, LOW modulation symbol rate $(R_{ms})_L$, LOW carrier tracking bandwidth B_φ, and the appropriate modulation removal loss factor η_φ. Calculation of the phase error variance σ^2_{pn} is, however, not as obvious here as in prior analyses. For the purpose of demodulation on the R/T data channel, the phase error variance $(\sigma^2_{pn})_D$ is due to phase noise in the frequency band $\{f_n$ to $(R_{ms})_D/2\}$ when f_n is the LOW PLL corner frequency and $(R_{ms})_D/2$ is one half the R/T PSK symbol rate. As discussed in Section 3, the upper frequency limitation is the result of using integrate and dump filtering which effectively suppresses high frequency phase reference estimation errors. On the other hand, the phase error variance $(\sigma^2_{pn})_L$ due to phase noise on the LOW is the result of phase noise in the frequency band $\{f_n$ to $(R_{ms})_L/2\}$. Therefore, if $(R_{ms})_L = (R_{ms})_D$ as in the lowest EDM

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* See Note 1 of the Annex.

R/T data rate then $(\sigma^2_{pn})_L = (\sigma^2_{pn})_D$. However, at higher R/T data rates $(R_{ms})_D \geq (R_{ms})_L$ which gives $(\sigma^2_{pn})_D \geq (\sigma^2_{pn})_L$ so that the total phase error variances for R/T data and LOW are such that $(\sigma^2_{tot})_D \geq (\sigma^2_{tot})_L$.

Thus, if one were to judge demodulation performance in the R/T channel, based solely upon carrier tracking performance on the LOW, severe errors could occur because of the failure to account for the additional phase noise in the frequency band $\left\{(R_{ms/2})_L \text{ to } (R_{ms/2})_D\right\}$.

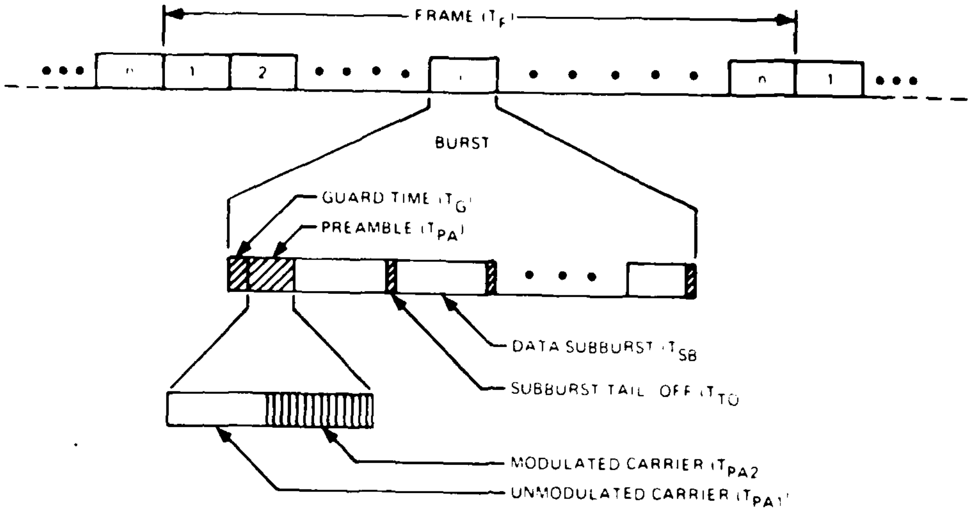
5.2.3 Performance of the USC-28 Operating with Terminals of the Phase II DSCS

A complete analysis of the USC-28 operating with an airborne AN/ASC-18 terminal is given in [1]. The reference gives a complete description of demodulation performance assuming that phase noise improvements are made to the AN/ASC-18 terminal. It was shown that the most critical performance requirements on carrier phase estimation performance (and, therefore R/T demodulation performance) occurred at the lowest R/T data rates, where the phase noise of the improved AN/ASC-18 terminal is similar to that of the cesium II curve of Figure 4-7. Since the cesium II phase noise is expected for HT-MT (AN/MSC-60) and upgraded MSC-46 terminals, performance at low data rates with these terminals will be similar to that shown in [1] for the AN/ASC-18 terminal. At high data rates, the cesium II phase noise performance is better than that of the improved AN/ASC-18; therefore, at high data rates performance of the USC-28 with the HT-MT and upgraded MSC-46 will be better than that shown in [1].

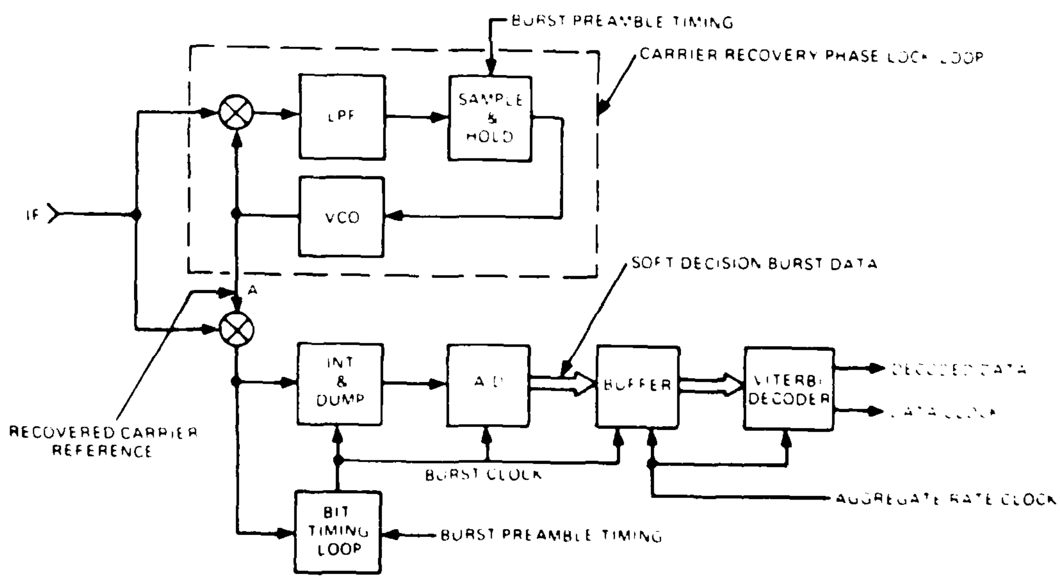
5.3 RAYTHEON INC. TDMA (EDM)

5.3.1 General[17]

The Raytheon EDM TDMA currently being procured by USASATCOMA is a burst coherent form of TDMA with a preamble preceding each data burst transmission which contains the residual carrier and bit timing references to maintain gated carrier and bit timing tracking loops in appropriate synchronization, within a certain minimum mean square phase error criterion. The frame structure and the basic carrier tracking demodulation and decoding techniques are depicted in Figure 5-6. Figure 5-6(a) shows the TDMA frame composed of n bursts originating from a network of n earth terminals each transmitting a burst of data. Each burst includes guard time, preamble time, and subbursts representing individual basebands and subburst tail-off time. The subburst tail-off time results from the desire to share a single error encoder and error decoder with time sequential subbursts of data. The single encoder must be "flushed" and returned to a reference condition before encoding the next subburst of data bits. This results in time in the burst that is unusable for sending data. The preamble is composed of a subburst of unmodulated carrier for carrier reference recovery and a subburst of modulated (alternate "ones" and "zeros") carrier for bit timing reference recovery. The functional demodulator detail necessary for understanding the problem is shown in Figure 5-6(b). TDMA frame, burst and subburst timing are derived (by other circuits not shown in Figure 5-6(b)) and made to gate tracking loops at the appropriat[illegible]es to permit "sampled data" burst coherent recovery of both carrier and bit timing references to enable efficient coherent demodulation of data subbursts. The recovered carrier reference multiplies the modulated [illegible]arrier at the appropriate times to demodulate the desired subbursts. This [illegible] is followed by matched filtering (integrate and dump) the noisy data [illegible] analog to digital conversion (for soft decision). The soft decision [illegible]red from burst rate down to an aggregate rate and decoded by a [illegible]



(a) FRAME, BURST AND PREAMBLE STRUCTURE



(b) FUNCTIONAL DEMODULATOR BLOCK DIAGRAM

Figure 5-6. TDMA Frame and Demodulator Details

5.3.2 Impact of Phase Noise Upon TDMA System Performance

One of the most useful measures of system performance for a TDMA system is TDMA frame efficiency η given by the following equation:

$$\eta = 1 - \frac{T_{OH}}{T_F} \qquad (5\text{-}1)$$

where T_{OH} = total frame overhead

T_F = total frame duration

However TDMA frame overhead is a function of many parameters as shown by the following equation:

$$T_{OH} = n\,T_g + \sum_{i=1}^{n} T_{PA}(i) + \sum_{i=1}^{n} T_{TO}(i) \qquad (5\text{-}2)$$

where T_g = guard time between bursts

$T_{PA}(i)$ = preamble time for i^{th} burst

$T_{TO}(i)$ = subburst tail-off time for i^{th} burst.

It may easily be seen that network size n and connectivity will have a profound influence on frame efficiency. In addition, for networks with various size terminals required preambles $T_{PA}(i)$ can be considerably different depending upon terminal G/T. Since the number of parameters which can be varied for this type of system is quite large and since our main interest in this paper is to indicate expected demodulation performance of a TDMA system operating in the presence of oscillator phase noise, the scope of the problem will be restricted by the following assumptions:

1. A maximum of 2-3 percent loss in frame efficiency is allocated to that part of preamble time reserved for residual carrier tracking.

2. The preceding frame efficiency loss is to be allocated equally between 20-30 earth terminals.

3. Demodulation losses will be based solely upon a 0.2-dB loss due to imperfect carrier phase tracking. All other demodulation losses including those due to symbol timing error are neglected as in the analysis of preceding sections.

4. Demodulation losses are based upon the assumption of soft decision (3 bit) rate 1/2, constraint length 7, Viterbi decoding followed by differential decoding. From the analysis of preceding sections and References 2 and 8, it is easily seen that carrier phase reference error variances of approximately -15 dB and ~ -28 dB are required for coded operation with BPSK and QPSK, respectively.

5. (F.R.) = 1200 frames per second

Assumptions 1 and 2 translate to a required duty factor of 0.001 for residual carrier tracking preamble time.

It has been shown [18] that a gated PLL will behave similar to a continuously tracking PLL if the effective loop time constant is much larger than the TDMA frame duration and if the gain in the gated PLL is increased by the duty factor d^{-1}. Mathematically this may be stated as:

$$\frac{(1 - d)}{F.R.} \quad \frac{\omega_n}{2\zeta} \quad << \quad 1 \qquad (5-3)$$

where

$$d = \frac{T_{CPA}}{T_F} = \text{carrier preamble duty factor}$$

T_{CPA} = carrier preamble time duration

and (ω_n, ξ) are the equivalent continuous PLL (natural radian frequency damping factor).

Therefore, Equation 3-10 (a, b) may be used to calculate PLL carrier tracking error variance provided the energy per symbol to noise (E_s/N_o) is replaced by its averaged value over the TDMA frame duration $(E_s/N_o)_{AV}$, where $(E_s/N_o)_{AV} = dT_s C/N_o$ where C is the received carrier power and T_s equals duration of each PSK symbol in the received burst.

Figures 5-7 through 5-12 (see also Tables D-25 through D-46) show expected demodulation performance for the Raytheon EDM TDMA system using a 100-Hz carrier tracking PLL bandwidth and optimum PLL bandwidth. Operation is assumed in the presence of oscillator phase noise contributed by two and three terminals of the following types:

1. *Modified* HT-MT (Figure 4-1)
2. "Cesium II" (Figure 4-7)
3. "Crystal II" (Figure 4-7).

The reader is reminded that all references to "bits" in the tables refer to "modulation bits" while in the figures the term "bits" refer to "information bits." Thus, due to the rate of 1/2 coding, the following relationships hold.

$$R_{ib} = R_{mb}/2$$

$$\text{and } E_{ib}/N_o = E_{mb}/N_o + 3 \text{ (dB)}$$

where

R_{ib} Information bit rate (information bps)

R_{mb} Modulation bit rate (modulation bps)

Tables S-3 (a) and (b) summarize the minimum and maximum allowable Raytheon TDMA data rates when used with possible phase noise contributions expected in the Phase II DSCS. Table S-3 (a) shows these results when a constant PLL noise bandwidth of 100 Hz (one sided) is used and Table S-3 (b) shows results when an optimum PLL bandwidth is chosen as a function of data rate. These tables (and Figures 5-7 through 5-12) show that dramatic improvements in performance are obtained when an optimum PLL noise bandwidth is used. They also show that the additional complexity of a variable bandwidth PLL is well justified based on demodulation improvements.

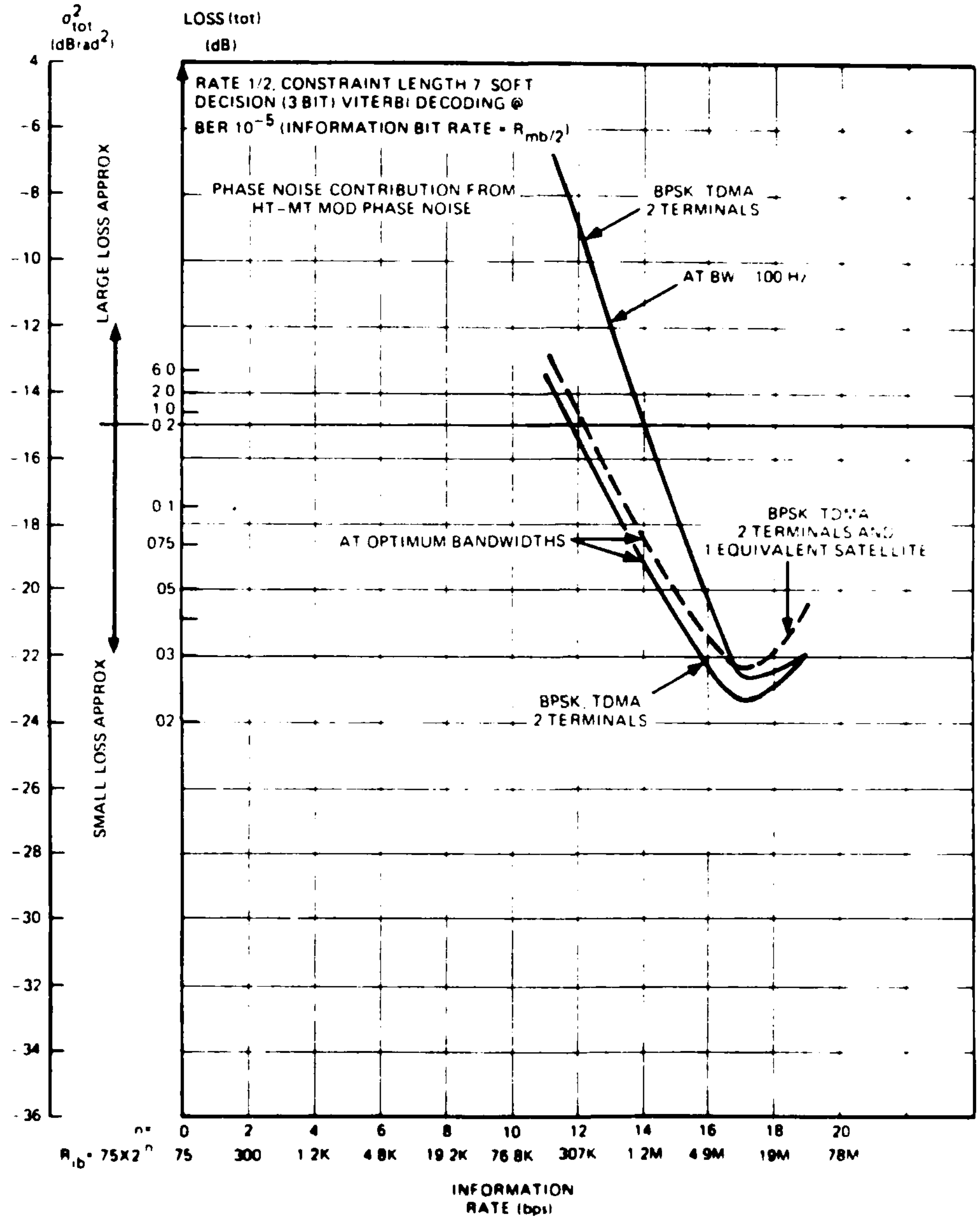


Figure 5-7. Raytheon Inc. BPSK TDMA Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2 tot, versus Information Bit Rate, R_{ib}(bps)

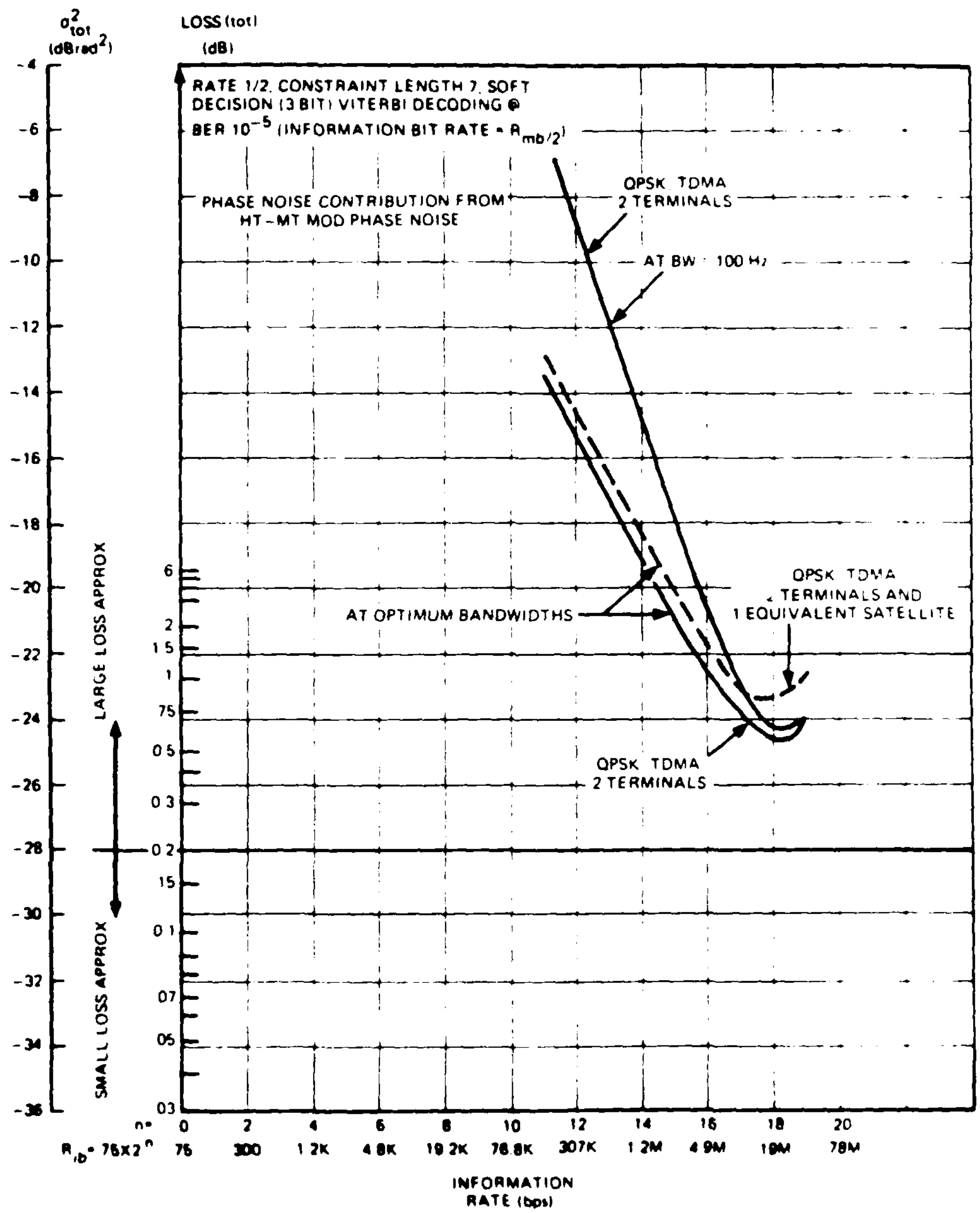


Figure 5-8. Raytheon Inc. QPSK TDMA Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps)

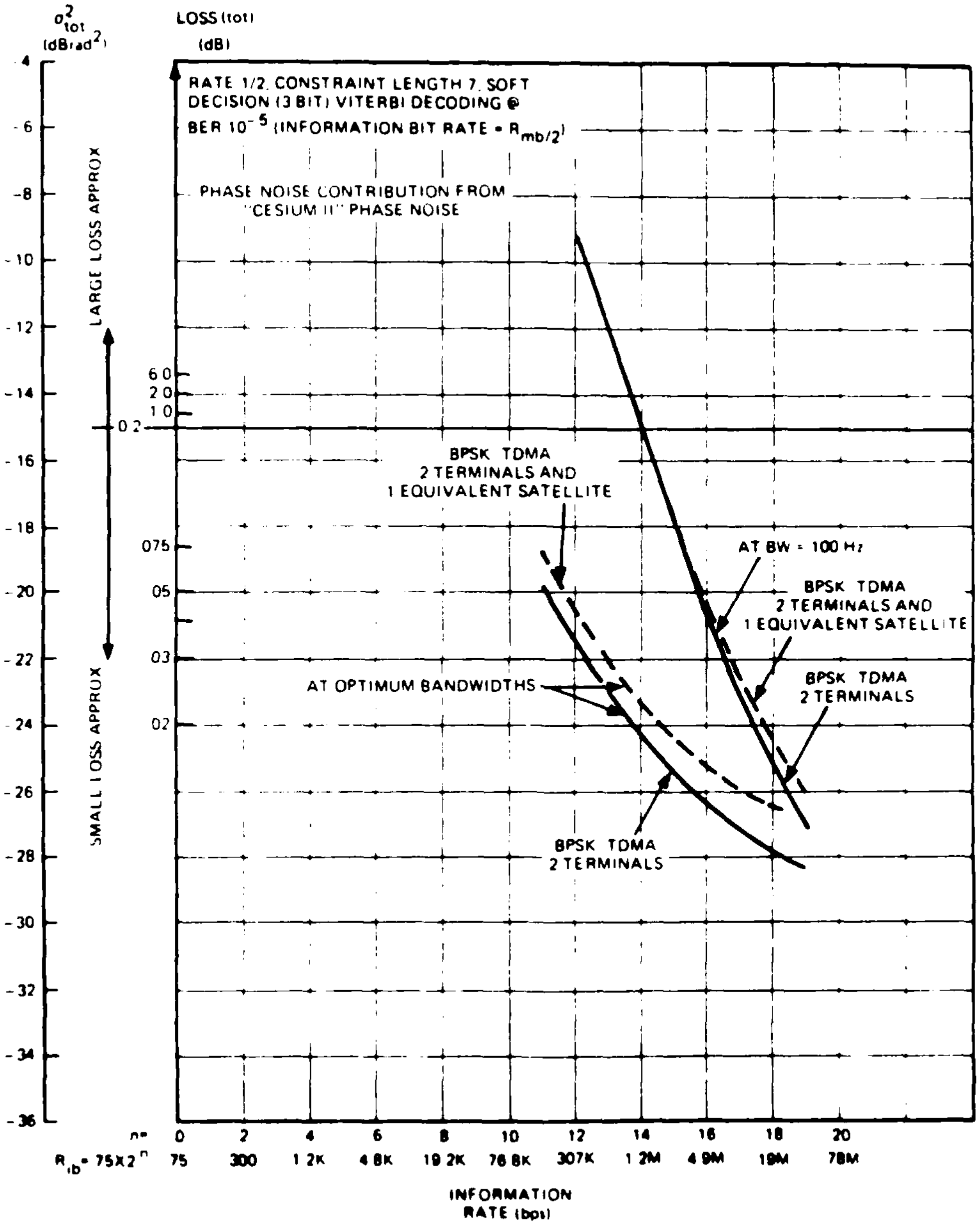


Figure 5-9. Raytheon Inc. BPSK TDMA Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps)

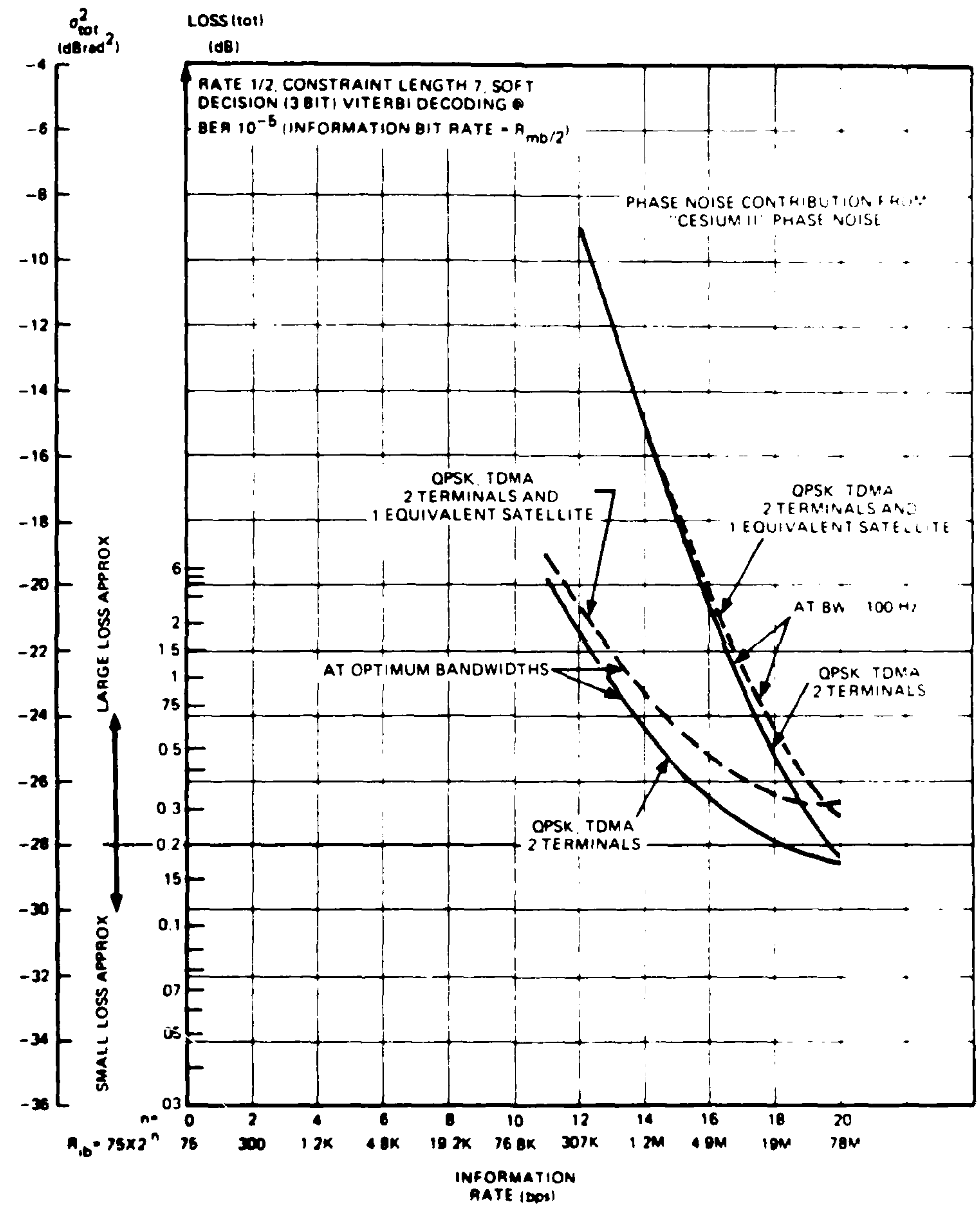


Figure 5-10. Raytheon Inc. QPSK TDMA Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps)

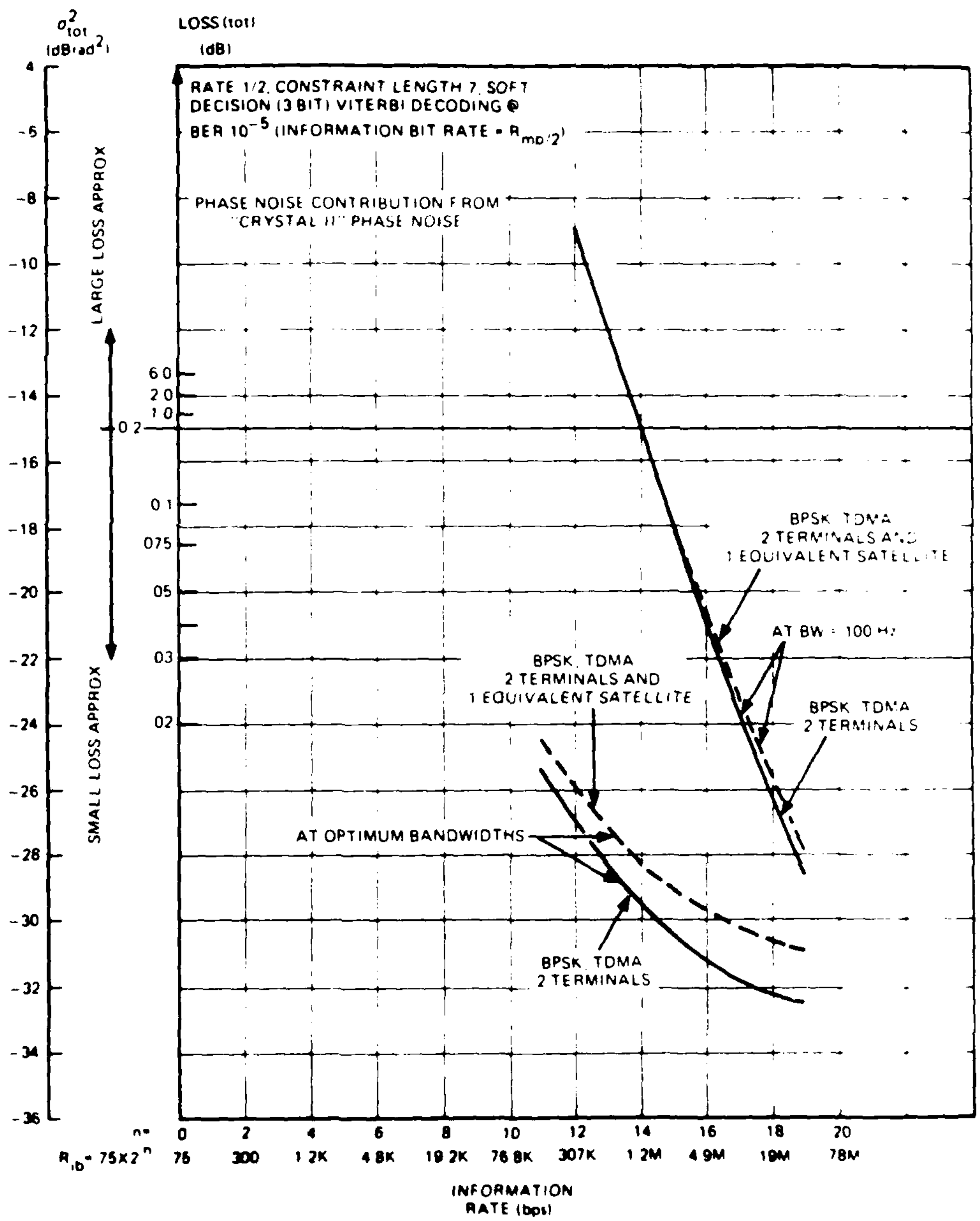


Figure 5-11. Raytheon Inc. BPSK TDMA Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps)

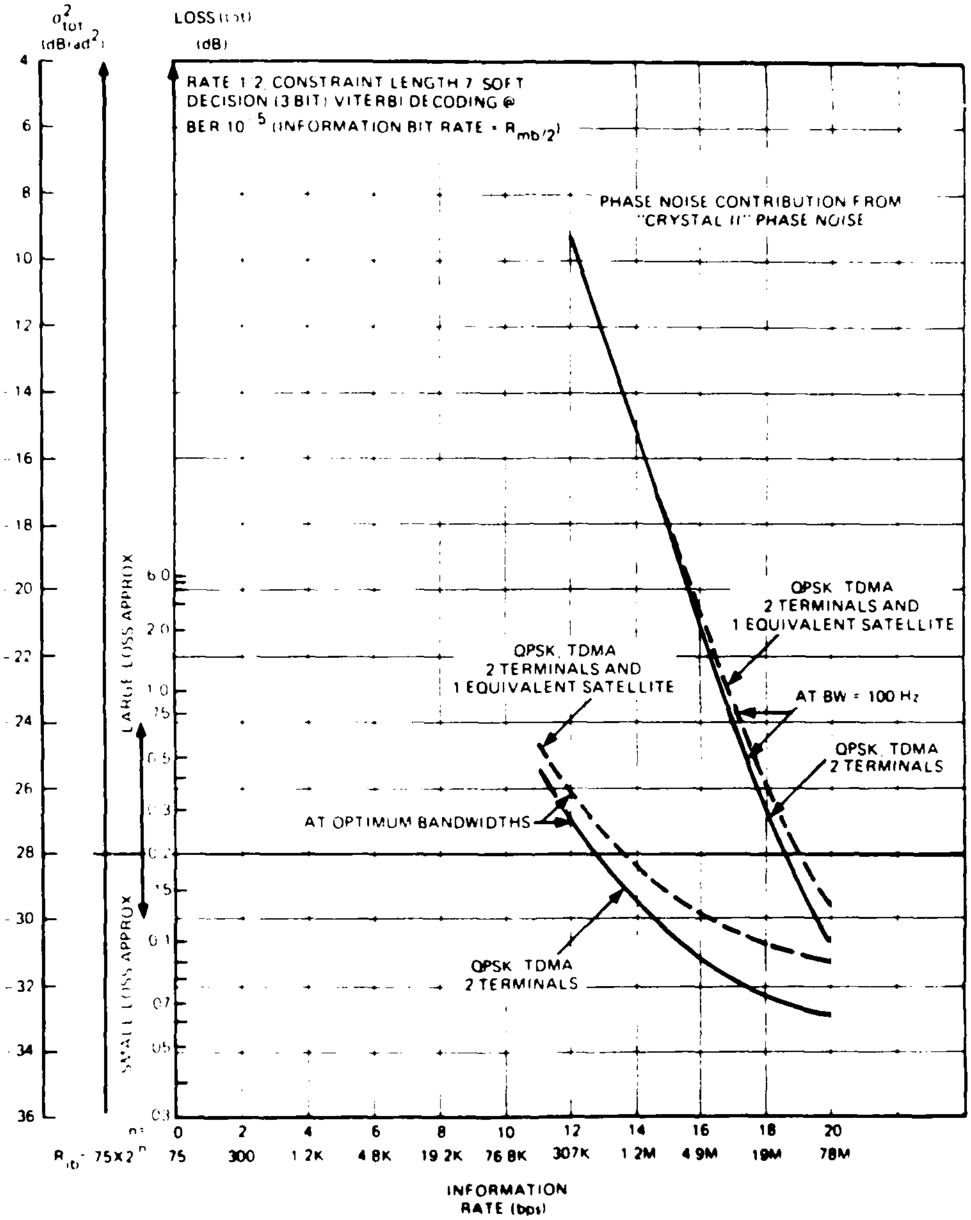


Figure 5-12. Raytheon Inc. QPSK TDMA Demodulation Loss (dB), L_{tot}, and Total Phase Variance, σ^2_{tot}, versus Information Bit Rate, R_{ib} (bps)

SECTION 6 - PHASE NOISE SPECIFICATIONS FOR TERMINALS OF THE DSCS

6.1 ANALYTICAL STUDY

All of the preceding performance analyses were based on the assumption of a known or specified oscillator phase noise spectral density. Because system performance results depend primarily on the area under the phase noise density curve between the PLL corner frequency and the PSK symbol rate, it is possible to specify an infinite number of phase noise spectra which will meet certain performance criteria. On the other hand, if we are to devise a phase noise specification that will ensure stated system performance measure, it should be remembered that the particular shape has only secondary influence on the performance. Only the total phase noise power from the corner frequency of the loop to one-half the symbol rate is of concern. Therefore, it is logical and convenient to set standards for the maximum phase noise power in a given band. Since most frequency sources show phase noise characteristics f^{-s} with the exponent s ranging from 1 to 3, the most critical characteristic f^{-3} may be assumed in determining the frequency band to be specified. Then, whatever the actual phase noise characteristic may be, the total phase noise variance can be met.

Stated more explicitly, if the phase noise specification is based on an f^{-s_1} phase noise characteristic about the loop corner frequency f_n and if f^{-s_2} phase noise is actually experienced, the total phase noise variance can be lowered relative to its specified value, provided $s_2 < s_1$. (We will prove this for s-values larger than 1 since we can infer about $s \leq 1$ by continuity.) It can be verified as follows. If the phase noise variance in the band specified by assuming f^{-s_1} phase noise equals the phase noise variance for f^{-s_2} phase noise then from Equation (3-31) we have

$$\frac{H_{s_1}}{(s_1-1)\, x_{s_1}^{s_1-1}} = L\, x_{s_1} \qquad \frac{H_{s_2}}{(s_2-1)\, x_{s_1}^{s_2-1}} = L\, x_{s_1} \quad (s_1 > s_2 > 1) \tag{6-1}$$

so

$$H_{s_2} = \frac{s_2-1}{s_1-1}\, H_{s_1}\, x_{s_1}^{s_2-s_1} \quad (s_1 > s_2 > 1) \tag{6-2}$$

Since the optimum x-value for an f^{-s_2} phase jitter characteristic is given by

$$x_{s_2} = \sqrt[s_2]{\frac{H_{s_2}}{L}} = \sqrt[s_2]{\frac{s_2-1}{s_1-1}}\; x_{s_1} \quad (s_1 > s_2 > 1) \tag{6-3}$$

the minimum total phase noise variance

$$\begin{aligned} \sigma^2_{s_2} &= \frac{s_2}{s_2-1}\, L\, x_{s_2} \quad (s_2 > 1) \\ &= \frac{s_2}{s_2-1} \sqrt[s_2]{\frac{s_2-1}{s_1-1}}\; L\, x_{s_1} \end{aligned} \tag{6-4}$$

is less than $\sigma^2_{s_1} = \frac{s_1}{s_1-1} L\, x_{s_1}$ for $s_2 < s_1$ because the function

$$\frac{\sigma^2_s}{\sigma^2_{s_1}} = f(s) = \frac{s}{s_1}\left(\frac{s_1-1}{s-1}\right)^{1-\frac{1}{s}} \quad (s_1 > s > 1) \tag{6-5}$$

is monotone and increasing for $s < s_1$.* Thus we can conclude that f^{-3} phase noise is more critical than for example f^{-1} or f^{-2}.

---

* It follows that f(s) has these properties because the derivative of f(s) is positive for $1 < s < s_1$ as shown in Appendix B.

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EFFECTS OF PHASE NOISE AND THERMAL NOISE UPON COHERENT PSK DEMO--ETC(U)
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Based on the assumption that we are faced with an f^{-3} phase noise characteristic, the resulting phase noise variance from the band f_0 to ∞ is given by

$$\frac{h_3}{2 f_0^2} \qquad \int_{f_0}^{\infty} \frac{h_3}{f^3}\, df \tag{6-6}$$

Equating this with the contribution due to f^{-3} phase noise in Equation (3-20), it implies that when using a PLL with a matched filter processor the lower band frequency should be set to

$$f_0 \qquad \frac{B_\varphi}{4.3} \tag{6-7}$$

Furthermore, if we set the upper band limit to R_s 2, the phase error variance, due to phase noise (power) in the band $(f_0, R_s$ 2), will give an upper bound to the minimum achievable phase noise variance according to Equation (3-20) (regardless of whether f^{-1}, f^{-2}, or f^{-3} phase noise is dominating).

From [2] and [8] it is known that to prevent more than 0.2 dB in equivalent power loss, due to phase noise at the decision point, it will be necessary that phase noise variance $\sigma^2_{tot} \approx -15$ dB from BPSK and σ^2_{tot} -28 dB for QPSK when coding is used. Assuming an f^{-3} phase noise characteristic, the system should be designed such that two-thirds of the total phase noise variance σ^2_{tot} is due to the phase-locked loop variance caused by the additive Gaussian noise. That is, the equivalent power loss requirement translates into the loop phase noise variances.

$$\sigma^2_{th} \qquad -16.77 \approx -17 \text{ dB for BPSK}$$

$$\sigma^2_{th} \qquad -29.77 \approx -30 \text{ dB for QPSK} \tag{6-8}$$

where according to Equation (3-9) we have

$$\sigma^2_{th} \qquad \frac{N_o B_\varphi}{E_s R_s} \eta_\varphi \tag{6-9}$$

At the operation point $E_s/N_o = E_{mb}/N_o = 1.3$ dB for rate 1/2 coded BPSK and $E_s/N_o = E_{mb}/N_o + 3 = 4.3$ dB for coded QPSK. Furthermore, from Table A-1 (taken from [8]) the values of the degradation factor η_φ are given both for decision-feedback (DF) and matched filter (MF power loop implementations). Thus, given σ^2_{th}, E_s/N_o and η_φ we can calculate the corresponding B_φ/R_s ratio using Equation (6-7) and, using $f_0 = B_\varphi/4.3$, determine the corresponding frequency specification band (f_0, $R_s/2$) for a given PSK symbol rate R_s. This has been carried out and the general algorithmic structure has been presented in Table 6-1 and a specific set of frequency bands was given in Table S-4 and Figure S-1. Note that the power loop implementation will specify wider bands, which means that a prescribed phase noise variance (power) requirement will be harder to meet. (The last band for R_s = 80 M symbols per second has been modified and extended down to 23 Hz since it refers to the TDMA operation. Use of a loop having a noise bandwidth of 100 Hz has been suggested.)[17]

Having determined the frequency bands that are related to the data rates, we must now determine the allowable phase noise power in these bands. In Table 1-5 the maximum phase noise power in a particular band is given for a set of equivalent power loss* values using the Gaussian loss approximation.[8] The table also distributes the total phase noise contributions on two and three terminals. The two-terminal case is applicable when the satellite has a negligible phase noise contribution, and it is assumed that the transmitting and receiving terminals have equal contributions. The three-terminal case assumes the satellite has a contribution equal to one of the terminals.

An example of the use of the preceding techniques for generating a phase noise specification is given in Paragraph 6.2. Phase noise specifications for the AN/MSC-60 (HT) "follow-on" and the AN/MSC-46 "upgrade" are given in Tables 1-6 and 1-7, respectively.

*A word of caution: It is impossible to satisfy demodulation loss criteria (minimum MSE) while simultaneously having inadequate carrier tracking loop cycle skipping performance. Criteria for these parameters must also be satisfied in any system analysis.

Table 6-1. Algorithm for Generating Frequency Specification Bands $[f_o, R_s/2]$

$\{R_s$ - PSK symbol rate$\}$

	$\frac{R_s}{2} / f_o$	
	BPSK	QPSK
Matched Filter Power Loop	110	2350
Decision Feedback	92	1435

6.2. APPLICATIONS

6.2.1 General

Terminal phase noise specification designed for BPSK systems in the DSCS are derived using the following assumptions:

1. Rate 1/2, constraint length 7, convolutional encoding with soft decision (3-bit) Viterbi decoding is used (see Table 1-5).
2. Maximum allowable demodulation loss to the imperfect carrier phase estimation is < 0.2 dB (see Table 1-5).
3. Equal phase noise contribution from terminal transmitter, terminal receiver and satellite (see Table 1-5).
4. Conservative case; systems will use matched filter power loops for carrier phase estimation (see Table 6-1).

6.2.2 BPSK System Specifications

In addition to the assumptions listed in the preceding section, specific equipments notably the Radiation BPSK MD-921G modem have the following PSK symbol rate limitations:

32 Ksps - 10 Msps

Using Table 6-1 and the assumptions in Paragraph 6.2.1, the following two frequency bands may be derived using the lowest and highest BPSK symbol rates:

(145 Hz - 16 kHz)

and

(45.5 kHz - 5 MHz)

That is, $\{ f_o, R_s/2 \}$ where $\frac{R_s}{2} \Big/ f_o = 110$.

Using Table 1-4 note that in each of the preceding bands the total phase noise added to any transmitted or received carrier should not exceed -24.5 dB rad^2 to keep demodulation losses, due to errors in carrier phase estimation, ≤ 0.2 dB. Since the above described modem may be used for any BPSK symbol rate within the stated limits, a complete specification would require additional overlapping specification bands. This composite phase noise specification (overlapping bands) may easily be satisfied by state of the art designs. Therefore a simplified, but slightly more stringent single band specification which is still easily achieved by state of the art techniques is given by the following:

<u>BPSK Specification</u>

> The total spurious content added to any transmitted or received carrier, including phase noise and discrete spurious signals from both sides of the carrier, shall be at least 25 dB below the carrier level when measured in a band 145 Hz to 5 MHz from the carrier frequency.

In the preceding specification an attempt has been made to use language and style suited to actual equipment specifications.

6.2.3 QPSK System Specifications

Techniques analogous to those used for BPSK are used to derive QPSK phase noise specifications.

For the DSCS, QPSK symbol rates are expected within the ranges:

(32 Ksps - 40 Msps)

From Table 6-1 and the assumptions of Paragraph 6.2.1 a series of overlapping specification bands (f_o, $R_s/2$) where $\frac{R_s}{2} / f_o$ 2850 may be derived.

Then using Table 1-4 and the same set of assumptions, the following specifications may be derived for QPSK signaling.

QPSK Specification

The total spurious content added to any transmitted or received carrier, including phase noise and discrete spurious signals from both sides of the carrier shall be at least 37.5 dB below the carrier level when measured in the following bands:

5 Hz to 16 Hz from the carrier frequency

20 Hz to 76 kHz from the carrier frequency

200 Hz to 0.6 MHz from the carrier frequency

1.7 kHz to 5 MHz from the carrier frequency

7 kHz to 20 MHz from the carrier frequency.

6.2.4 Phase Noise Specifications for MRL's USC-28 BPSK Spread Spectrum

The USC-28 may essentially be treated as a modified BPSK system after the PN sequence has been removed as discussed in Paragraph 5.2.3 of this report. Carrier phase estimates are derived from the LOW. Worst case operation (from a phase noise point of view) is a rate 1/2 coded LOW at 150 BPSK symbols per second and an E_s/N_o = 1.3 dB (energy per modulation symbol noise density). If the R/T channel operates at 150 BPSK symbols per second with an E_s/N_o = 1.3 dB the carrier phase estimation problem is equivalent to that for an ordinary 150 BPSK symbol rate system.

Thus the following frequency band (f_o, $R_s/2$) derived as in Paragraph 6.2.2 (.6 - 75 Hz) where $\frac{R_s}{2} \Big/ f_o = 110$.

Using Table 1-4 and the assumption of Paragraph 6.2.1, the total phase noise added to any transmitted or received carrier should not exceed -24.5 dB $rad^2 \simeq -25$ dB rad^2 to keep demodulation losses due to errors in carrier phase estimation $\leq$ 0.2 dB.

As noted in Paragraph 5.2.3, even at high data rates the carrier phase estimate was derived from the LOW which was constrained (in rate 1/2 coded operation) to operate at a 150 BPSK symbol per second rate. Operation at the highest BPSK symbol rate (5 msps) means that the effective phase noise band is stretched to become (0.6 Hz-2.5 MHz). However, operation of the USC-28 is such that at the high data rates so much additional power is added to the LOW (E_s/N_o >>1.3 dB) that losses due to thermally induced errors in carrier phase estimation become negligible. Thus, from Table S-5 the total error variance σ_{tot}^2 = -15 dB rad^2 for a 0.2 dB demodulation loss may be assumed to be caused by untracked oscillator phase noise and distributed equally (1/3 allocation or -4.77 dB) to give -19.77 rad^2 for each of the up- and down-converters and the satellite. Thus, the following terminal phase noise specification is generated for terminals operating with the USC-28 BPSK spread spectrum system.

<u>Specification for USC-28</u>

The total spurious content added to any transmitted or received carrier, including phase noise and discrete spurious signals, shall not exceed conditions specified in the following paragraphs.

1. Total spurious content from both sides of the carrier shall be at least 25 dB below the carrier level when measured in a band 0.6 Hz to 75 Hz from the carrier frequency.

2. Total spurious content from both sides of the carrier shall be at least 20 dB below the carrier level when measured in a band 0.6 Hz to 2.5 MHz from the carrier frequency.

6.2.5 Phase Noise Specifications for Raytheon Inc.'s Burst Coherent TDMA

As discussed in Paragraph 5.3 of this report the 100 Hz carrier tracking PLL bandwidth for the TDMA system is constrained to much less than the TDMA frame rate independent of the actual BPSK or QPSK symbol rate. The

most critical (i.e., largest) frequency band may therefore be derived by using the highest QPSK symbol rate of 80 msps and Equation (6-7). The TDMA band is thus (23 Hz-40 MHz) or $\{B_{\varphi}/4.3,\ R_s/2\}$ where B_{φ} 100 Hz.

Using Table S-4 the following specification is derived for QPSK TDMA where demodulation losses due to imperfect carrier phase estimation are ≤ 0.2 dB.

Phase Noise Specifications for Raytheon Inc. TDMA

> The total spurious content added to any transmitted or received carrier including phase noise and discrete spurious signal from both sides of the carrier shall be at least 37.5 dB below the carrier level when measured in a band 23 Hz-40 MHz from the carrier frequency.

6.2.6 Summary

In Paragraph 6.2 phase noise specifications have been presented for various equipment expected to be operational in the DSCS. Since the AN/MSC-60 (HT) follow-on earth terminal is expected to work with all or some modified version of the preceding equipment, all of the specifications of Paragraph 6.2 must be equalled or exceeded by this earth terminal.

Deleting all but the most stringent specifications gives the proposed specification on phase noise for the follow-on AN/MSC-60 (HT) earth terminal shown in Table S-6.

6.3 PHASE NOISE SPECIFICATIONS FOR THE AN/MSC-46 "UPGRADE"

Preceding derivations of phase noise specifications for the AN/MSC-60 (HT) follow-on included all contributions to phase noise on the transmitted or received signal including the effects of the frequency standard which is the basic source of all frequencies in the terminal. However, the AN/MSC-46 upgrade will be based on terminal designs for which the frequency standard

will be government furnished equipment (GFE) procured under separate contract.

Therefore a sub-system phase noise specification must be generated for terminal designs excluding the effects of a frequency standard. Of course these subsystem phase noise specifications must be consistent with total phase noise specifications on terminals with a frequency standard.

Considerable difficulty is experienced when attempting to allocate phase noise between the terminal itself and its frequency standard. This difficulty occurs in spite of the fact that phase noise due to the standard dominates at very low frequencies while phase noise due to the terminal dominates at higher frequencies because the crossover frequency between these two phase noise sources is a function of very specific equipment designs.

However, discussions with both a Fluke representative and Comtech proved that Fluke's 6160 A/AO synthesizer has a 3-pole 200 Hz low pass filter which filters phase noise due to the frequency standard beyond this point. Since the Fluke synthesizer is an integral part of the AN/MSC-46 upgrade design we can state that phase noise due to the standard will dominate at frequencies below 200 Hz while phase noise due to the terminal itself will dominate at frequencies above 200 Hz.

The terminal phase noise specifications for the AN/MSC-46 upgrade shown in Table S-7 is seen to be a modification of the phase noise specifications for the AN/MSC-60 (HT) follow-on (Table S-6) only within the region below 200 Hz. A comparison of these phase noise specifications in the frequency band (0.6 Hz-75 Hz) indicates that the phase noise contribution due to the "terminal only" must be 12 dB below that due to the frequency standard, i.e., less than 0.25 dB additional phase noise caused by the "terminal only." A second band has also been derived in the same manner to cover the frequency band (1.8 Hz-200 Hz).

In summary, phase noise specifications have been derived for terminal designs which do not include a frequency standard (as in Table S-7 for the upgraded AN/MSC-46). To meet total system phase noise specifications, frequency standards must be chosen that satisfy the total specification. At this time a complete set of phase noise specifications have not been derived for the frequency standard independent of the terminal design. Thus, for the present, frequency standards are best evaluated in conjunction with a specific terminal design.

ANNEX

NOTE 1:

In this paper the term Soft Viterbi decoding represents the following set of operational values.

It is assumed that rate 1/2, constraint length 7, convolutional encoding is preceded by a differential encoding process as shown in Figure 2-1. On the receiver side it is assumed that soft Viterbi (8-level) decoding is performed and then followed by differential decoding. The nominal BER at the system output is assumed to be 10^{-5} required an energy per information bit/noise density $E_{ib}/N_o = 4.3$ dB. Due to the rate 1/2 structure of the encoding process this corresponds to an energy per modulation bit/noise density E_{mb}/N_o 1.3 dB.

NOTE 2:

An additional degradation factor[9] should also be included for decision feedback loops since the phase error at the symbol decision point causes an increase in the number of erroneous symbol decisions which directly change the loop gain by $(1 - 2P_s \sin^2 \frac{\pi}{M})$ where P_s is the symbol error probability and M is the type of PSK modulation.

Loop corner frequency f_n as defined by Equations (3-11) through (3-14) is proportional to the square root of loop gain and therefore f_n should be modified as:

$$f_n' = f_n \sqrt{1 - 2P_s \sin^2 \frac{\pi}{M}}$$

For small values of P_s this effect on the corner frequency may be neglected.

NOTE 3:

It is assumed that the B5400 crystal would eventually be phase-locked to an atomic standard to prevent long-term frequency drifts of the crystal

oscillator. However, in this report it is assumed that the bandwidth at which the crystal is phase-locked to the atomic standard would be considerably smaller (< a factor of 10) than the optimum bandwidth of the receiver tracking loops. Under these conditions the effects of phase noise in the atomic standard may be neglected as in the curve labelled "crystal II" of Figure 4-7. Of course, the analysis in this report could easily be used to indicate expected performance should the appropriate data become available.

APPENDIX A – PARTIALLY COHERENT M-ARY PSK DEMODULATION LOSS FUNCTIONS

In [6] it is shown that the variance of the phase estimate obtained using a power loop tracking on M-ARY PSK signal in the presence of additive white Gaussian noise (AWGN) is given by:

$$\sigma^2 = \frac{N_o B_\varphi}{E_s R_s} \eta_\varphi \tag{A-1}$$

where

$$\eta_\varphi = \eta_\varphi^{(P)} = \frac{1}{M^2} \sum_{k=1}^{M} \binom{M}{k}^2 k! \left(\frac{N_o}{E_s}\right)^{k-1} \tag{A-2}$$

In [8] it is shown that the variance of the phase estimate obtained using decision feedback (DF) tracking an M-ARY PSK signal in the presence of AWGN is also given by Equation (A-1) where:

$$\eta_\varphi = \eta_\varphi^{(d)} = \frac{1 - 2P_s \frac{E_s}{N_o} \left[\sin^2\left(\frac{2\pi}{M}\right) - \frac{4N_o}{3E_s} \sin^2\left(\frac{\pi}{M}\right)\right]}{\left[1 - 2P_s \sin^2\left(\frac{\pi}{M}\right)\right]^2} \tag{A-3*}$$

P_s = modulation symbol error probability (i.e., the probability that the symbol is received correctly is $1 - P_s$) and all other parameters are as defined in Sections 2 and 3.

Evaluation of (A-2) and (A-3) is provided in Table A-1 for various configurations of interest. Also shown in Table A-1 are tabulated loss functions for symbol timing loops which have not been considered in this report.

---

* This correction factor differs slightly from the one given in [9]. The difference lies in the precise definition of loop bandwidth. The above form is preferred.

Table A-1. Performance Comparison of Decision-Feedback and Power Loop Implementations for M-ary PSK Demodulation

```
DECISION FEEDBACK IMPLEMENTATION
     ETA1 = CARRIER PHASE ESTIMATION LOSS
     ETA3 = SYMBOL TIMING ESTIMATION LOSS

POWER LOOP IMPLEMENTATION
     ETA2 = CARRIER PHASE ESTIMATION LOSS
     ETA4 = SYMBOL TIMING ESTIMATION LOSS
```

M = 2   0   INT. TIME/SYM. DURATION = .5

EB/NO(DB)	ETA1(DB)	ETA2(DB)	ETA3(DB)	ETA4(DB)
-1.7	1.66578	2.40438	1.66578	4.36[illegible]
0	1.00541	1.76091	1.00541	3.5218[illegible]
.3	.906782	1.6632	.906782	3.39124
1.3	.618655	1.36928	.618655	2.99139
2	.454478	1.19084	.454478	2.74379
4	.146925	.788386	.146925	2.16085
6	2.77372E-2	.513819	2.77372E-2	1.7647[illegible]
8	2.21137E-3	.331199	2.21137E-3	1.48754
10	4.47504E-5	.211893	4.47504E-5	1.30334
12	9.70724E-8	.134894	9.70724E-8	1.18296
14	0	8.55987E-2	0	1.10596

M = 3   0   INT. TIME/SYM. DURATION = .5

EB/NO(DB)	ETA1(DB)	ETA2(DB)	ETA3(DB)	ETA4(DB)
-1.7	2.95208	5.37444	1.48598	3.39096
0	2.01728	4.02647	.776707	2.74373
.3	1.86647	3.81796	.679196	2.66579
1.3	1.39662	3.18308	.411143	2.34951
2	1.10049	2.79172	.272396	2.14981
4	.437178	1.88764	5.61809E-2	1.76432
6	9.62211E-2	1.25061	3.54861E-3	1.48751
8	7.68836E-3	.815819	-2.26599E-4	1.30330
10	1.21179E-4	.526323	-9.22188E-6	1.18295
12	9.70724E-8	.336943	0	1.10525
14	0	.214606	0	1.0555

M = 4   0   INT. TIME/SYM. DURATION = .5

EB/NO(DB)	ETA1(DB)	ETA2(DB)	ETA3(DB)	ETA4(DB)
-1.7	3.93778	9.1468	1.54657	2.98756
0	3.19466	6.93507	.961851	2.47038
.3	3.05112	6.58834	.871336	2.34676
1.3	2.54861	5.50503	.602143	2.09641
2	2.17949	4.86276	.445566	1.94667
4	1.12841	3.31489	.145993	1.61384
6	.346735	2.20837	2.77043E-2	1.38[illegible]
8	4.38679E-2	1.44590	2.21111E-3	1.23465
10	1.38973E-3	.934873	4.47504E-5	1.13943
12	4.95069E-6	.599359	9.70724E-8	1.07735
14	0	.38208	0	1.03771

M = 8   0   INT. TIME/SYM. DURATION = .5

EB/NO(DB)	ETA1(DB)	ETA2(DB)	ETA3(DB)	ETA4(DB)
-1.7	3.73902	29.9389	2.078	2.41336
0	3.92623	23.5407	2.09903	1.9957[illegible]
.3	3.94059	22.5017	2.0927	1.9342[illegible]
1.3	3.93765	19.2408	2.04415	1.7535[illegible]
2	3.88305	17.1462	1.98197	1.66668
4	3.44418	12.024	1.65913	1.40773
6	2.54931	8.14105	1.17432	1.25088
8	1.35037	5.35972	.544935	1.14888
10	.383453	3.46161	.182894	1.08339
12	3.94079E-2	2.2105[illegible]	1.41815E-2	1.04154
14	8.63977E-4	1.40311	3.08277E-4	1.01496

In Section 3 of this report we have shown how carrier phase estimation quality (MSE) could be described in terms of phase error variances which depend upon various system parameters (e.g., E_{ib}/N_o, $S_{\delta\phi}(f)$, B_φ, η_φ, etc.). In the preceding we have also summarized how the modulation removal loss factor η_φ (required in the calculation of phase error variance) may be calculated for matched filter power loops and decision feedback loops. It remains, however, to indicate how these phase error variances may be translated into demodulation losses from ideal performance. In [8] it is shown that for small demodulation losses, the following equation will provide an accurate description of M-ary PSK demodulation loss L in dB versus phase error variance in radians.

$$L = 4.34\,\sigma^2 \left\{ 1 + \frac{2E_{mb}}{N_o} \log_2 M \left[\cos^2\left(\frac{\pi}{M}\right) + \frac{\sigma^2}{2}\left(1 - 3\cos^2\left(\frac{\pi}{M}\right)\right)\right]\right\} \qquad \text{(A-4)}$$

where

$\frac{E_{mb}}{N_o}$ = energy per modulation bit/noise density expressed in a pure number.

For BPSK and QPSK this result simplifies to:

<u>BPSK (M = 2)</u>

$$L = 4.34\,\sigma^2 \left(1 + \sigma^2 \frac{E_{mb}}{N_o}\right) \text{(dB)} \qquad \text{(A-5)}$$

<u>QPSK (M = 4)</u>

$$L = 4.34\,\sigma^2 \left(1 + \frac{2E_b}{N_o} - \sigma^2 \frac{E_{mb}}{N_o}\right) \text{(dB)} \qquad \text{(A-6)}$$

and if $\sigma^2 << 1$ the following familiar forms result:

<u>BPSK (M = 2)</u>

$$L = 4.34\,\sigma^2 \text{ (dB)} \tag{A-7}$$

<u>QPSK (M = 4)</u>

$$L = 4.34\,\sigma^2 \left(1 + \frac{2E_b}{N_o}\right) \text{ (dB)} \tag{A-8}$$

Equations (A-4) through (A-8) will provide accurate loss estimates when the carrier phase estimation error variance σ^2 is of sufficiently small magnitude. This accuracy limitation occurs because the preceding equations are based upon an assumption of a Gaussian phase error density as being an accurate characterization of the phase error process in a second order carrier phase estimator. Actually it is known from [5] that even for a first order simple PLL, the Gaussian assumption is only valid at small σ^2 values (high signal-to-noise-ratios) and that a Tikhonov phase error density is exact for a first order loop in the presence of AWGN and is also a good approximation for a second order loop. The Tikhonov phase error density is given by the following

$$p(\varphi) = \exp(\alpha \cos\varphi)/2\pi I_o(\alpha) \qquad |\varphi| \leq \pi \tag{A-9}$$

where α is the PLL signal-to-noise ratio.

Charles Wolfson has assumed that a modified form of the Tikhonov density may be used to describe the phase error process in various power loops (i.e., modulation removal loops such as squaring, quadrupling) to derive demodulation losses for BPSK and QPSK systems. This modified Tikhonov phase error density is given by the following:

$$p(M\varphi) = M \exp[\alpha_M \cos(M\varphi)]/2\pi I_o(\alpha_M) \tag{A-10}$$

where

$$|\varphi| \leq \pi/M \text{ ,}$$

$$M^2 \alpha_M = \sigma^{-2}$$

and M is the maximum number of signal phases.

Since the modified Tikhonov density will approach a Gaussian density for small σ^2 (large α), losses based upon either technique are in good agreement when demodulation losses are small. In [8] it is shown that results based upon the Gaussian approximation are accurate to within 0.01 dB when

$$L < 0.66 \left(\frac{E_s}{N_o}\right)^{-2/3} \text{ (dB)} \quad M = 2 \tag{A-11}$$

and

$$L < 0.4 \text{ (dB)} \quad M = 4 \tag{A-12}$$

For the large loss case, it is obvious that neither the Gaussian nor modified Tikhonov density will accurately describe the phase error process in a modulation removal PLL. However, it is believed that the approximation based upon the Tikhonov density will provide the most accurate description of demodulation losses currently available. Due to the nature of the analyses used to derive demodulation losses in [8] and [2, 7] based, respectively, on the Gaussian or Tikhonov densities, it is believed that the former will provide the most accurate characterization for small losses (small σ^2) while the latter

will provide the most accurate characterization for large losses (large σ^2). Therefore, in this report a two part (large and small) loss approximation is used to indicate demodulation performance.

In the preceding we have limited our discussion of demodulation losses to PSK systems which are unencoded. To access the impact of convolutional encoding and Viterbi decoding (as described in Note 1 of the Annex to this report) we may use the coder functional as described in [19].

That is;*

$$P_e \text{ (coding)} = \text{"const"}\, \Phi\left(-\sqrt{\frac{2E_b}{N_o}\, d_{min}}\right) \tag{A-13}$$

where

$d_{min} = 10$ is the minimum free distance of the rate 1/2, constraint 7 convolutional code.

Equation (A-13) implies that the net effect of coding is to increasing the effective signal-to-noise ratio by 10 log d_{min} in the error function. Thus, in the Gaussian approximation to the loss function (Equations (A-4) through (A-[illegible])) the equivalent losses for the coded case may be calculated with an effective E_b/N_o

$(E_b/N_o)_{eff} = E_b/N_o + 10$ dB.

For example, when using the Gaussian approximation and the system described in Note 1, and when L = 0.2 dB

$\sigma^2 = -14.9$ dB BPSK

$\sigma^2 = -27.8$ dB QPSK

---

* $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)\, dt$

The "const" in Equation (A-13) is not strictly a constant with respect to E_b/N_o but is much less dependent than the error integral Φ.

Results using the Tikhonov phase error density as described above and in [2] and [7] are summarized in Figure A-1.

The following convention has been adopted when plotting all of the (non-linear) demodulation loss scales shown in this report:

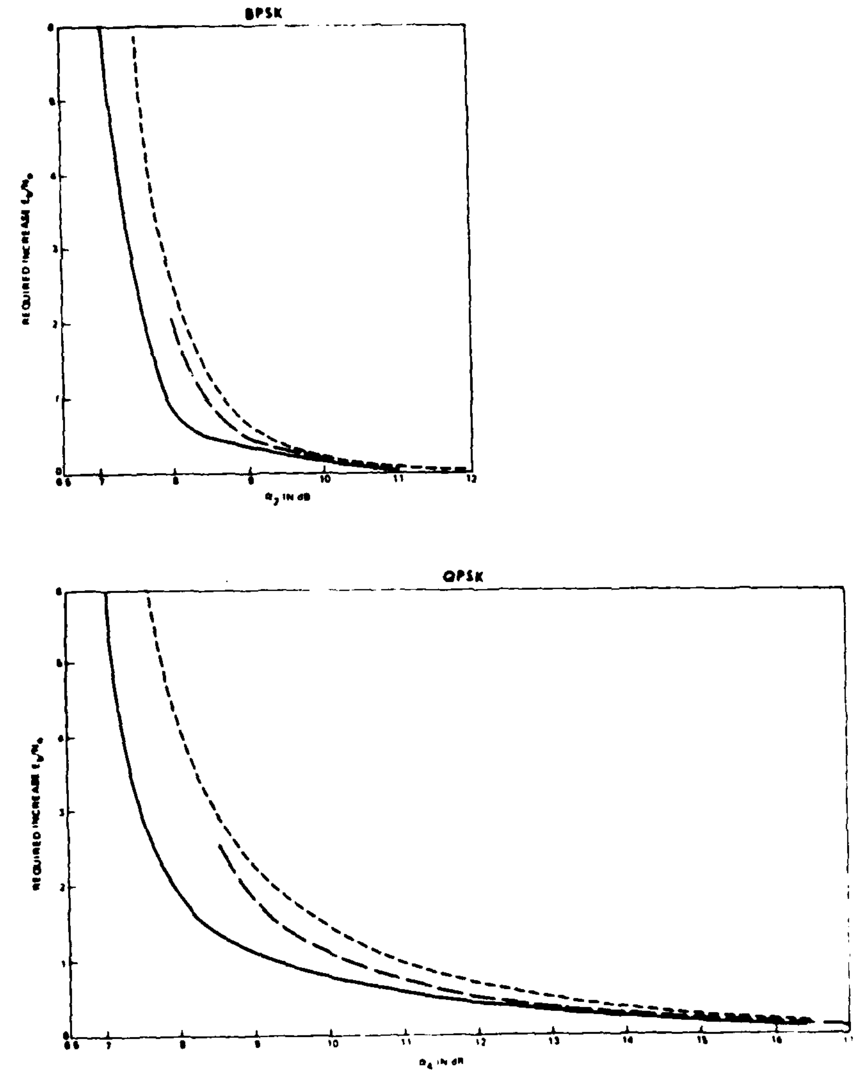
> When demodulation losses are less than 0.2 dB, then losses are based upon the Gaussian approximation. When the losses are > 0.2 dB then losses are based upon the Tikhonov approximation.

In our view this two part loss functional with a break point at 0.2 dB represents the best estimate of demodulation losses currently for the coded case.

More recently in [21], loss formulas were derived which indicate that the coded QPSK loss functional (both large and small loss approximation) used in this report may be too conservative. However, if new loss functionals are indeed proved to be more accurate than those used here (especially for QPSK), it is a simple matter to replot the loss ordinates of the demodulation performance curves of this report since the remainder of the analysis will remain affected.

SUPPRESSED CARRIER SYSTEMS

P (ε) = 10^{-5}



UNCODED
HARD DECISION DEMODULATION (Q=2)
SOFT DECISION DEMODULATION (Q=8)

Figure A-1. Comparison of Degradation Incurred for Auxiliary Carrier, Suppressed Carrier BPSK and QPSK Systems With and Without Viterbi Decoding

APPENDIX B – VERIFICATION OF WORST CASE PHASE NOISE ASSUMPTION

Verify that Equation (6-5)

$$\frac{\sigma_s^2}{\sigma_{s_1}^2} = f(s) = \frac{s}{s_1}\left(\frac{s_1-1}{s-1}\right)^{1-\frac{1}{s}} \qquad (s_1 \geq s > 1) \qquad (6\text{-}5)$$

is monotone and increasing for $(1<s\leq s_1)$. That is $f(s) \leq f(s_1)$ when $1< s \leq s_1$.

That $f(s)$ is monotone and increasing may be seen from the positiveness of the derivative of $\ln f(s)$ $(s_1 \geq s > 1)$. Now,

$$\ln f(s) = \ln \frac{s}{s_1} + \left(\frac{s-1}{s}\right) \ln \left(\frac{s_1-1}{s-1}\right) \qquad s_1 \geq s > 1 \qquad (B\text{-}1)$$

so

$$\frac{\partial \ln f(s)}{\partial s} = \frac{1}{s^2} \ln \frac{s_1-1}{s-1} \qquad (B\text{-}2)$$

Since $(s_1-1) > (s-1)$ when $(s_1 \geq s > 1)$

$$\ln \left(\frac{s_1-1}{s-1}\right) > 0 \qquad (B\text{-}3)$$

and

$$\frac{\partial \ln f(s)}{\partial s} > 0 \qquad (B\text{-}4)$$

Hence, we can conclude that $f(s) \leq f(s_1)$ for $1<s \leq s_1$.

APPENDIX C - EVALUATION OF INTEGRALS ASSOCIATED WITH PHASE NOISE INFLUENCE ON COHERENT PSK DEMODULATION

This appendix presents an evaluation of the integrals

$$I_k = \int_0^\infty \frac{x^k}{1+x^4} \left(\frac{\sin \alpha x}{\alpha x}\right)^2 dx \; ; \qquad \text{for } k = 0,\ 1,\ 2,\ 3,\ 4; \tag{C-1}$$

associated with phase noise influence on coherent PSK demodulation.

Although these integrals are very similar for all k-values, a closed form solution can only be obtained for even k-values using complex integration or residue calculus. For odd k-values we must settle for approximate integral evaluations.

C.1 EVEN K-VALUES

For even k-values, i.e., k = 0, 2, 4, we first rewrite I_k as

$$I_k = \frac{1}{\alpha^2} \int_0^\infty \frac{x^{k-2}}{1+x^4} \sin^2 \alpha x \; dx \tag{C-2}$$

For k = 0,

$$I_0 = \frac{1}{\alpha^2} \int_0^\infty \left(\frac{1}{x^2} - \frac{x^2}{1+x^4}\right) \sin^2 \alpha x \, dx \tag{C-3}$$

$$= \frac{1}{\alpha^2} \int_0^\infty \frac{\sin^2 \alpha x}{x^2} dx - \frac{1}{2\alpha^2} \int_0^\infty \frac{x^2}{1+x^4} (1-\cos 2\alpha x) \, dx \tag{C-4}$$

while for k = 2 or 4,

$$I_k = \frac{1}{2\alpha^2} \int_0^\infty \frac{x^{k-2}}{1+x^4} (1-\cos 2\alpha x) \; dx \tag{C-5}$$

Since the integrands of all four integrals are even functions, we can evaluate them over the interval $(-\infty, \infty)$ and obtain twice their values. This method makes it possible to determine the value of a particular integral as the residues of its integrand. Consequently, these integrals are also available in tables such as those published by [20], from which we directly obtain

$$\int_0^\infty \frac{\sin^2 \alpha x}{x^2}\, dx = \frac{\alpha \pi}{2} \tag{C-6}$$

$$\int_0^\infty \frac{\cos 2\alpha x}{1 + x^4}\, dx = \frac{\pi\sqrt{2}}{4}\, e^{-\sqrt{2}\alpha} (\cos \sqrt{2}\alpha - \sin \sqrt{2}\alpha) \tag{C-7}$$

$$\int_0^\infty \frac{x^2}{1 + x^2} \cos 2\alpha x\, dx = \frac{\pi\sqrt{2}}{4}\, e^{-\sqrt{2}\alpha} (\cos \sqrt{2}\alpha - \sin \sqrt{2}\alpha) \tag{C-8}$$

and, by letting $\alpha \to 0$ in Equations (C-7) and (C-8),

$$\int_0^\infty \frac{dx}{1 - x^4} = \int_0^\infty \frac{x^2}{1 - x^4}\, dx = \frac{\pi\sqrt{2}}{4} \tag{C-9}$$

Thus we get

$$
\begin{aligned}
I_o &= \frac{\alpha \pi}{2\alpha^2} - \frac{\pi\sqrt{2}}{8\alpha^2} \left[1 - e^{-\sqrt{2}\alpha} (\cos \sqrt{2}\alpha - \sin \sqrt{2}\alpha) \right] \\
&= \frac{\pi}{2\alpha} \left[1 - \frac{\sqrt{2}}{4\alpha} \left(2\sqrt{2}\alpha - 2\alpha^2 + \frac{2}{3}\alpha^4 + \ldots . \right) \right] \\
&= \frac{\pi}{2\alpha} \left(\frac{\sqrt{2}}{2}\alpha - \frac{\sqrt{2}}{6}\alpha^3 \ldots \right) \\
&= \frac{\pi\sqrt{2}}{4} \left(1 - \frac{1}{3}\alpha^2 + \ldots . \right)
\end{aligned} \tag{C-10}
$$

$$I_2 = \frac{1}{2\alpha^2} \; \frac{\pi\sqrt{2}}{4} \; \left[1 - e^{-\sqrt{2}\alpha} (\cos \sqrt{2}\alpha + \sin \sqrt{2}\alpha)\right]$$

$$= \frac{1}{2\alpha^2} \; \frac{\pi\sqrt{2}}{4} \; (2\alpha^2 - \frac{4}{3}\sqrt{2}\,\alpha^3 + \frac{2}{3}\alpha^4 + \ldots.)$$

$$= \frac{\pi\sqrt{2}}{4} \; (1 - \frac{2\sqrt{2}}{3}\,\alpha + \frac{1}{3}\alpha^2 + \ldots) \qquad \text{(C-11)}$$

and

$$I_4 = \frac{1}{2\alpha^2} \; \frac{\pi\sqrt{2}}{4} \; \left[1 - e^{-\sqrt{2}\alpha} (\cos \sqrt{2}\alpha - \sin \sqrt{2}\alpha)\right]$$

$$= \frac{\pi\sqrt{2}}{8\alpha^2} \; (2\sqrt{2}\,\alpha - 2\alpha^2 + \frac{2}{3}\alpha^4 + \ldots)$$

$$= \frac{\pi}{2} \; (\frac{1}{\alpha} - \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{2}}\;\alpha^2 + \ldots) \qquad \text{(C-12)}$$

For $\alpha << 1$, e.g., $\alpha < 0.1$, we can approximate

$$I_0 = \frac{\pi\sqrt{2}}{4} \qquad \text{(C-13)}$$

$$I_2 = \frac{\pi\sqrt{2}}{4} \; (1 - \frac{2\sqrt{2}}{3}\,\alpha) \qquad \text{(C-14)}$$

$$I_4 = \frac{\pi}{2} \; (\frac{1}{\alpha} - \frac{1}{\sqrt{2}}) \qquad \text{(C-15)}$$

C.2 ODD K-VALUES

For odd k-values, i.e., $k = 1, 3$, we must settle for approximate evaluations of the values of I_k. Since we desire to evaluate I_k for small α-values, we must first bound the integral

$$I_k = \int_0^\infty \frac{x^k}{1 + x^4} \left(\frac{\sin \alpha x}{\alpha x}\right)^2 dx \qquad \text{(C-16)}$$

with an upper bound

$$I_k = \int_0^a \frac{x^k}{1+x^4}\, dx + \frac{1}{\alpha^2} \int_a^\infty \frac{\sin^2 \alpha x}{x^{6-k}}\, dx \qquad \text{(C-17)}$$

and a lower bound ($|\alpha a| < \pi$)

$$I_k = \left(\frac{\sin \alpha a}{\alpha a}\right)^2 \int_0^a \frac{x^k}{1+x^4}\, dx + \frac{a^4}{1+a^4}\,\frac{1}{\alpha^2} \int_a^\infty \frac{\sin^2 \alpha x}{x^{6-k}}\, dx \qquad \text{(C-18)}$$

where the value of a will be chosen later to obtain tight bounds. The inequalities result from the observations that

$$\left(\frac{\sin \alpha a}{\alpha a}\right)^2 \leq \left(\frac{\sin \alpha x}{\alpha x}\right)^2 \leq 1 \qquad \text{for } 0 \leq |\alpha x| < |\alpha a| \leq \pi$$

and

$$\frac{a^4}{1+a^4} \leq \frac{x^4}{1+x^4} \leq 1 \qquad \text{for } x > a$$

Of the two remaining integrals, the first is elementary for k = 1, 3. We have

$$\int_0^a \frac{x\, dx}{1+x^4} = \frac{1}{2} \arctan a^2 \qquad \text{(C-19)}$$

$$\int_0^a \frac{x^3\, dx}{1+x^4} = \frac{1}{4} \ln\,(1+a^4) \qquad \text{(C-20)}$$

The other integral can be reduced to a simpler form and expressed in terms of the cosine integral ci(x). Since

$$\frac{1}{\alpha^2} \int_a^{\infty} \frac{\sin^2 \alpha x}{x^{6-k}} \, dx = \alpha^{3-k} \int_{\alpha a}^{\infty} \frac{\sin^2 u}{u^{6-k}} \, du \tag{C-21}$$

and using repeated partial integration with $\beta = \alpha a$,

$$\int_{\beta}^{\infty} \frac{\sin^2 u}{u^5} \, du = \frac{1}{4} \left(\frac{\sin \beta}{\beta}\right)^2 + \frac{1}{3\beta^2} \frac{\sin 2\beta}{2\beta} + \frac{1}{6} \frac{1}{\beta^2} - \frac{2}{3} \int_{\beta}^{\infty} \frac{\sin^2 u}{u^3} \, du \tag{C-22}$$

we arrive at

$$\int_{\beta}^{\infty} \frac{\sin^2 u}{u^3} \, du = \frac{1}{2} \left(\frac{\sin \beta}{\beta}\right)^2 + \frac{\sin 2\beta}{2\beta} - \mathrm{ci}\,(2\beta) \tag{C-23}$$

where

$$\mathrm{ci}(2\beta) \triangleq - \int_{2\beta}^{\infty} \frac{\cos u}{u} \, du = \gamma + \ln(2\beta) + \sum_{k=1}^{\infty} \frac{(-1)^k (2\beta)^{2k}}{2k\,(2k)!} \tag{C-24}$$

and $\gamma = 0.577\,215\ldots$ is Euler's constant. Thus, we have an upper bound for

$$I_1 = \frac{1}{2} \arctan a^2 - \frac{1}{a^2} \left[\frac{1}{4} \left(\frac{\sin \alpha a}{\alpha a}\right)^2 + \frac{1}{3} \frac{\sin 2\alpha a}{2\alpha a} + \frac{1}{6} \right]$$

$$- \frac{2}{3} \alpha^2 \left[\frac{1}{2} \left(\frac{\sin \alpha a}{\alpha a}\right)^2 + \frac{\sin 2\alpha a}{2\alpha a} - \mathrm{ci}\,(2\alpha a) \right] \tag{C-25}$$

Keeping the dominating terms for small α, we get

$$I_1 = \frac{1}{2} \arctan a^2 - \frac{1}{a^2} \left(\frac{3}{4} - \frac{11}{36} \alpha^2 a^2\right) - \left(1 - \frac{2}{3}\gamma\right)\alpha^2 + \frac{2}{3} \alpha^2 \ln 2\alpha a$$

$$= \frac{1}{2} \arctan a^2 - \frac{3}{4a^2} + \frac{2}{3}\alpha^2 \ln 2\alpha a - \left(\frac{47}{36} - \frac{2}{3}\gamma\right)\alpha^2 \tag{C-26}$$

The corresponding lower bound for

$$I_1 = \frac{1}{2} \left(\frac{\sin \alpha a}{\alpha a}\right)^2 \arctan a^2 + \frac{a^4}{1 + a^4} \left(\frac{3}{4a^2} - \frac{2}{3} \alpha^2 \ln 2 \alpha a + \dots\right) \quad \text{(C-27)}$$

If we choose a = 2.5, the upper and lower bound for $\alpha < 0.1$ will be within 2 percent of each other. Thus, the integral I_1, with good accuracy, is approximately equal to

$$I_1 = \frac{\pi}{4} + 0.04 - \frac{2}{3} \alpha^2 \ln \alpha + 0.15 \alpha^2 = 0.825 - \frac{2}{3} \alpha^2 \ln \alpha \quad (\alpha < 0.1) \quad \text{(C-28)}$$

For k = 3 we have the upper bound

$$I_3 = \frac{1}{4} \ln (1 + a^4) + \frac{1}{2} \left(\frac{\sin \alpha a}{\alpha a}\right)^2 + \frac{\sin 2 \alpha a}{2 \alpha a} - \mathrm{ci}\,(2 \alpha a)$$

$$= \frac{1}{4} \ln (1 + a^4) + \left(\frac{3}{2} - \gamma\right) - \ln 2 \alpha a + \frac{1}{6} \alpha^2 a^2 + \dots \quad \text{(C-29)}$$

and the lower bound

$$I_3 = \frac{1}{4} \ln (1 + a^4) \left(\frac{\sin \alpha a}{\alpha a}\right)^2 + \frac{a^4}{1 + a^4} \left[\left(\frac{3}{2} - \gamma\right) - \ln 2 \alpha a + \frac{1}{6} \alpha^2 a^2 + \dots\right] \quad \text{(C-30)}$$

Again, with a = 2.5, the upper bound is within 2 percent of the true value of I_3 for α-values less than 0.1. Thus we have the approximation

$$I_3 = \ln \frac{1}{\alpha} + 0.235 + 1.04 \alpha^2 \quad (\alpha < .1) \quad \text{(C-31)}$$

APPENDIX D - TABULATED RESULTS

Table D-1. BPSK Decision Feedback With HT-MT Mod Phase Noise (2 Terminals) (Losses-Soft Decision Viterbi Rate 1/2 Decoding BER 10^{-5})

OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.[illegible]E-10 RAD/HZ H1= 0 RAD
H2= .01 RAD*HZ H3= .2 RAD*HZ↑2

M-ARY PSK M= 2 EB/NO= 1.3 DB

Mod. BIT RATE* B/S	BW-OPT(DB) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(PN) DB
75	6.86532	-9.10945	-11.0654	-13.5151
150	8.64972	-11.067	-13.0722	-15.3874
300	10.8979	-13.0126	-15.0791	-17.2304
1200	17.2985	-16.8563	-19.0931	-20.8085
4800	27.4543	-20.613	-23.1076	-24.2085
19200	43.5474	-24.2458	-27.1247	-27.3916
76800	68.91	-27.703	-31.1521	-30.3147
307200	107.913	-30.8574	-35.2248	-32.8352
1228800	157.07	-33.0366	-39.6152	-34.1148
4915200	504.219	-33.2059	-40.5706	-34.0861
19660800	967.665	-28.7913	-43.7601	-28.9319
78643200	1890.59	-23.0145	-46.872	-23.0324
CODING GAIN OF		10	DB	

DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN < .116469 DB

Mod. BIT RATE* B/S	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS (TOT) (DB)	TIKHONOV APPROX. LOSS (TH) (DB)
75	1.41504	.698032	>6	>6
150	.697623	.356174	>6	>6
300	.363093	.191214	>6	.25
1200	.114408	6.03682E-2	.12	.1
4800	4.21016E-2	2.26185E-2		
19200	1.71557E-2	8.63432E-3		
76800	7.52394E-3	3.36315E-3		
307200	3.60192E-3	1.30847E-3		
1228800	2.17138E-3	4.74904E-4		
4915200	2.08781E-3	3.81019E-4		
19660800	5.83489E-3	1.82694E-4		
78643200	[illegible].31398E-2	8.92096E-5		

* Information bit rate = $\frac{1}{2}$ · Modulation bit rate

Table D-2. QPSK Decision Feedback With HT-MT Phase Noise (2 Terminals) (Losses-Soft Decision Viterbi Rate 1/2 Decoding BER = 10^{-5})

```
OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.26E-10  RAD/HZ                H1= 0  RAD
H2= .01  RAD*HZ                     H3= .2  RAD*HZ↑2

M-ARY PSK    M= 4                   EB/NO= 1.3  DB
```

Mod. BIT RATE * B/S	BL-OPT(TOT) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(PN) DB
75	5.9201	-7.84914	-9.77871	-12.3015
150	7.45884	-9.81322	-11.7856	-14.1901
300	9.3975	-11.7668	-13.7925	-16.0524
1200	14.9171	-15.6322	-17.8063	-19.6789
4800	23.6766	-19.4203	-21.8206	-23.1405
19200	37.5656	-23.0976	-25.8365	-26.3974
76800	59.5106	-26.6198	-29.8589	-29.4128
307200	93.6815	-29.9086	-33.9091	-32.113
1228800	145.543	-32.6466	-38.1067	-34.1001
4915200	411.24	-34.3505	-39.5258	-35.9226
19660800	783.254	-31.4106	-42.7478	-31.7421
78643200	1523.05	-25.9719	-45.8808	-26.0165

CODING GAIN = [illegible] 1[illegible] DB

```
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN <    .4            DB
```

Mod. BIT RATE * B/S	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH) (DB)
75	18.2492	12.1295	>6	>6
150	12.0286	7.79221	>6	>6
300	7.85491	4.96873	>6	>6
1200	3.47601	1.99622	>6	>6
4800	1.38005	.79595	>6	1.6
19200	.592644	.316323	.9	.35
76800	.264173	.125377	.26	<.1
307200	.123953	4.93554E-2		
1228800	.066001	1.87767E-2		
4915200	4.45861E-2	1.35431E-2		
19660800	.087712	6.4496E-3		
78643200	.300621	3.12498E-3		

* Information bit rate = ½ · modulation bit rate

Table D-3. BPSK Power Loop With HT-MT Mod Phase Noise (2 Terminals) (Losses-Soft Decision Viterbi Rate 1/2 Decoding BER = 10^{-5})

```
OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.06E-10  RAD/HZ              H1= 0  RAD
H2= .01  RAD*HZ                   H3= .2  RAD*HZ↑2

M-ARY PSK   M= 2                  EB/N0= 1.3  DB
```

Mod. BIT RATE * B/S	BL-OPT(MB) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(PN) DB
75	6.48097	-8.61975	-10.5649	-13.0442
150	8.16548	-10.5799	-12.5718	-14.9231
300	10.2878	-12.5287	-14.5787	-16.7738
1200	16.3302	-16.3812	-18.5926	-20.3713
4800	25.9184	-20.1505	-22.6071	-23.7956
19200	41.1161	-23.801	-26.6236	-27.0074
76800	65.0959	-27.2824	-30.6488	-29.9637
307200	103.179	-30.4767	-34.7113	-32.533
1228800	152.217	-32.7906	-39.0009	-33.9786
4915200	445.634	-33.0638	-40.1657	-34.0053
19660800	891.113	-28.7663	-43.3674	-28.9195
78643200	1737.89	-23.0113	-46.4871	-23.0308

```
CODING GAIN OF                    10       DB
DEMODULATION LOSSES USING GAUSSIAN
(APPROX ARE ACCURATE WHEN<     .116469     DB
```

Mod. BIT RATE * (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH (DB)
75	1.70184	.381063	>6	>6
150	.827981	.240054	>6	>6
300	.425149	.151223	>6	1.2
1200	.130846	6.00106E-2	.13	<.02
4800	4.73841E-2	2.38114E-2		
19200	1.91051E-2	9.44341E-3		
76800	8.31883E-3	3.73775E-3		
307200	3.93583E-3	1.46676E-3		
1228800	2.29878E-3	5.46261E-4		
4915200	2.15772E-3	4.17754E-4		
19660800	5.86914E-3	1.99871E-4		
78643200	2.21581E-2	9.74493E-5		

* Information bit rate = ½ · modulation bit rate

Table D-4. QPSK Power Loop With HT-MT Mod Phase Noise (2 Terminals) (Losses-Soft Decision Viterbi Rate 1/2 Decoding BER = 10^{-5})

OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.26E-10 RAD/HZ H1= 0 RAD
H2= .01 RAD*HZ H3= .2 RAD*HZ↑2

M-ARY PSK M= 4 EB/NO= 1.3 DB

Mod. BIT RATE * (B/S)	BL-OPT(MB) (HZ)	PH-VAR(TOT) (DB)	PH-VAR(TH) (DB)	PH-VAR(PN) (DB)
75	4.71103	-5.89877	-7.79442	-10.4124
150	5.93551	-7.8713	-9.8013	-12.3229
300	7.47456	-9.83527	-11.8082	-14.2112
1200	11.5708	-13.7291	-15.822	-17.9041
4800	18.8426	-17.5591	-19.836	-21.4524
19200	29.9033	-21.2958	-23.8508	-24.8147
76800	47.4311	-24.8997	-27.8689	-27.9516
307200	74.9680	-28.3065	-31.9004	-30.8024
1228800	116.868	-31.2964	-35.9928	-33.0953
4915200	302.065	-33.3714	-37.8894	-35.2647
19660800	567.891	-31.1543	-41.1683	-31.6103
78643200	1094.14	-25.9356	-44.3408	-25.9987

CODING GAIN OF 10 DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN< .4 DB

Mod. BIT RATE * (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH) (DB)
75	27.3509	18.5616	>6	>6
150	18.2639	14.0699	>6	>6
300	11.9809	7.75312	>6	>6
1200	5.04019	3.13772	>6	>6
4800	2.11216	1.25472	>6	>6
19200	.897825	.499325	1.9	.7
76800	.392352	.198197	.5	.2
307200	.179213	7.83704E-2	.13	<.1
1228800	9.00588E-2	3.05487E-2		
4915200	5.58587E-2	1.97403E-2		
19660800	9.20537E-2	9.27848E-3		
78643200	.309196	4.46924E-3		

* Information bit rate = $\frac{1}{2}$ · modulation bit rate

Table D-5. BPSK Decision Feedback With HT-MT Mod Phase Noise (3 Terminals) (Losses-Soft Decision Viterbi Rate 1/2 Decoding BER = 10^{-5})

OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.89E-10 RAD/HZ   H1= 0 RAD
H2= .015 RAD*HZ   H3= .3 RAD*HZ↑2

M-ARY PSK M= 2   EB/NO= 1.3 DB

Mod. BIT RATE * B/S	BW-OPT(DB) HZ	PH-VAR(TOT) DB	PH-VAR(TH) HZ	PH-VAR(PN) DB
75	7.85881	-8.49497	-10.4784	-12.8527
150	9.90142	-10.4458	-12.4853	-14.7084
300	12.4749	-12.3831	-14.4922	-16.532
1200	19.8012	-16.2047	-18.5062	-20.0623
4800	31.4233	-19.9294	-22.5212	-23.4029
19200	49.8225	-23.5182	-26.54	-26.5169
76800	78.713	-26.9146	-30.5745	-29.36
307200	122.287	-29.9649	-34.6818	-31.7532
1228800	322.904	-32.6829	-36.4868	-35.0225
4915200	608.708	-31.7458	-39.7527	-32.4939
19660800	1175.3	-27.0813	-42.9159	-27.1961
78643200	2305.02	-21.2602	-46.0112	-21.2747
CODING GAIN OF		10	DB	

DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN < .116469 DB

Mod. BIT RATE * B/S	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH) (DB)
75	1.78459	.858422	>6	>6
150	.868445	.431279	>6	>6
300	.446085	.228235	>6	1.2
1200	.137615	7.28637E-2	.13	<.1
4800	5.01591E-2	2.61199E-2		
19200	2.04635E-2	9.91499E-3		
76800	9.0738E-3	3.84718E-3		
307200	4.43473E-3	1.48355E-3		
1228800	2.35694E-3	9.77522E-4		
4915200	2.92961E-3	4.6009E-4		
19660800	8.7234E-3	2.21923E-4		
78643200	3.57463E-2	1.08771E-4		

* Information bit rate = ½ · modulation bit rate

Table D-6. QPSK Decision Feedback With HT-Mt Mod Phase Noise (3 Terminals) (Losses-Soft Decision Viterbi Rate 1/2 Decoding BER = 10^{-5})

OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.89E-10 RAD/HZ   H1= 0 RAD
H2= .015 RAD*HZ   H3= .3 RAD*HZ↑2

M-ARY PSK   M= 4   EB/NO= 1.3 DB

Mod. BIT RATE * B/S	BW-OPT(DB) HZ	PH-VAR(TOT) DB	PH-VAR(TH) HZ	PH-VAR(PN) DB
75	6.77682	-7.23829	-9.19174	-11.6483
150	8.53822	-9.19649	-11.1986	-13.5221
300	10.7574	-11.1428	-13.2055	-15.3668
1200	17.0755	-14.9886	-17.2195	-18.9494
4800	27.1007	-18.7482	-21.234	-22.3551
19200	42.9877	-22.3856	-25.251	-25.5459
76800	68.0327	-25.8537	-29.2778	-28.4862
307200	106.6	-29.0617	-33.348	-31.0871
1228800	156.149	-31.5718	-37.7108	-32.7824
4915200	495.241	-33.0989	-38.7186	-34.4905
19660800	979.847	-29.768	-41.9109	-30.0416
78643200	1855.04	-24.2273	-45.0245	-24.2636

CODING GAIN OF   10   DB

DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN <   .4   DB

Mod. BIT RATE * B/S	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH) (DB)
75	20.8464	13.7774	>6	>6
150	13.7633	8.87722	>6	>6
300	8.98766	5.67084	>6	>6
1200	3.79123	2.28239	>6	>6
4800	1.60956	.910642	>6	1.9
19200	.699124	.361914	1.25	.4
76800	.315072	.143317	.35	.1
307200	.150622	5.61599E-2		
1228800	8.45276E-2	2.05688E-2		
4915200	5.94753E-2	1.63093E-2		
19660800	.128028	7.82035E-3		
78643200	.457934	3.81833E-3		

* Information bit rate = ½ · modulation bit rate

Table D-7. BPSK Power Loop With HT-MT Mod Phase Noise (3 Terminals) (Losses-Soft Decision Viterbi Rate 1/2 Decoding BER = 10^{-5})

```
OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.89E-10  RAD/HZ            H1= 0   RAD
H2= .015  RAD*HZ                H3= .3   RAD*HZ↑2

M-ARY PSK   M= 2                EB/NO= 1.3   DB
```

Mod. BIT RATE * B/S	BW-OPT(MB) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(PN) DB
75	7.41887	-8.00671	-9.97797	-12.3855
150	9.34711	-9.96054	-11.9849	-14.2485
300	11.7765	-11.9015	-13.9918	-16.0806
1200	18.6929	-15.7328	-18.0058	-19.6317
4800	29.6659	-19.4715	-22.0206	-22.998
19200	47.0457	-23.0795	-26.0385	-26.1417
76800	74.3857	-26.5025	-30.0696	-29.0194
307200	116.012	-29.5993	-34.1599	-31.4694
1228800	153.686	-31.418	-38.9592	-32.2595
4915200	561.606	-31.623	-39.3518	-32.4254
19660800	1081.65	-27.0608	-42.5259	-27.186
78643200	2118.06	-21.2575	-45.628	-21.2734

```
CODING GAIN OF                   10      DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN<        .116669      DB
```

Mod. BIT RATE * (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH) (DB)
75	2.15283	.436207	>6	>6
150	1.03415	.274792	>6	>6
300	.524008	.173106	>6	.75
1200	.157711	6.86931E-2	.25	<.1
4800	5.64837E-2	2.72543E-2	<.1	<.1
19200	2.27746E-2	1.08053E-2		
76800	1.00035E-2	4.27099E-3		
307200	4.82993E-3	1.66532E-3		
1228800	3.1615E-3	5.51533E-4		
4915200	3.01441E-3	5.03858E-4		
19660800	8.76567E-3	2.46607E-4		
78643200	3.57699E-2	1.18767E-4		

* Information bit rate = ½ · modulation bit rate

Table D-8. QPSK Power Loop With HT-MT Mod Phase Noise (3 Terminals) (Losses-Soft Decision Viterbi Rate 1/2 Decoding BER = 10^{-5})

```
OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.90E-10  RAD/HZ                H1= 0  RAD
H2= .015  RAD*HZ                    H3= .3  RAD*HZ↑2

M-ARY PSK   M= 4                    EB/NO= 1.3  DB
```

Mod. BIT RATE * B/S	BW-OPT(MR) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(PN) DB
75	5.29278	-5.29265	-7.20746	-9.77152
150	6.79446	-7.26039	-9.21433	-11.6695
300	8.56045	-9.21846	-11.2212	-13.543
1200	12.5886	-13.0962	-15.235	-17.1978
4800	21.5685	-16.9053	-19.2492	-20.696
19200	34.255	-20.605	-23.2645	-23.9969
76800	54.2471	-24.162	-27.2848	-27.0627
307200	85.5762	-27.5014	-31.3256	-29.8265
1228800	131.912	-30.3539	-35.4669	-31.9535
4915200	362.234	-32.2076	-37.1005	-33.909
19660800	686.472	-29.5544	-40.3447	-29.9324
78643200	1330.06	-24.1977	-43.4929	-24.2491

```
CODING GAIN OF                         10          DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN<              .4          DB
```

Mod. BIT RATE * (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH) (DB)
75	30.7809	20.98	>6	>6
150	20.7511	13.7103	>6	>6
300	13.6981	8.8329	>6	>6
1200	5.81193	3.58511	>6	>6
4800	2.45358	1.4352	>6	>6
19200	1.05196	.571325	2.6	.95
76800	.464856	.226701	.6	.25
307200	.215681	8.94554E-2	.2	<.1
1228800	.111878	3.44804E-2		
4915200	7.30192E-2	.023672		
19660800	.13448	1.12158E-2		
78643200	.461063	5.43587E-3		

* Information bit rate = ½ · modulation bit rate

Table D-9. Demodulation Performance - BPSK, Power Loop, "Cesium II" Phase Noise (2 Terminals)

{Losses - Soft Decision (3 bit), R=$\frac{1}{2}$, K=7, Viterbi Decoding @ BER=10^{-5}}

* Information bit rate = $\frac{1}{2}$ · modulation bit rate

```
PHASE VARIANCES VS. DATA RATE *
AT OPTIMUM LOOP BANDWIDTH:

M-ARY PSK     M= 2

FH/NO= 1.3 ...

POWER LOOP IMPLEMENTATION

DAMPING FACTOR=  .707107
```

DMR (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(OS) (DB-RAD**2)
75.	1.407	-15.3288	-17.1475	-19.8922
300.	2.272	-19.0659	-21.1386	-23.2473
1200.	3.654	-22.5083	-25.0465	-25.9687
4800.	5.895	-25.4811	-29.0388	-28.0968
19200.	9.278	-27.5888	-33.0890	-29.0237
76800.	14.807	-28.6160	-37.0635	-29.2877
307200.	56.222	-28.7551	-37.3059	-29.4081
1228800.	384.101	-30.8038	-34.9811	-32.5760
4915200.	757.037	-32.6264	-38.0550	-34.0926
19660800.	1195.178	-33.0356	-42.0924	-34.6561
78643200.	1884.044	-34.5741	-46.1364	-34.8883

```
DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

 SENSITIVITY GAIN (DB) =  10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN <  .116649 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR=  .707107
```

DMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
75.	1.407	.1774E+00	.1040E+00
300.	2.272	.6298E-01	.3686E-01
1200.	3.654	.2620E-01	.1349E-01
4800.	5.895	.1275E-01	.5507E-02
19200.	9.278	.7743E-02	.2145E-02
76800.	14.807	.6080E-02	.8576E-03
307200.	56.222	.5895E-02	.8091E-03
1228800.	384.101	.3821E-02	.1394E-02
4915200.	757.037	.2388E-02	.6806E-03
19660800.	1195.178	.1763E-02	.2683E-03
78643200.	1884.044	.1521E-02	.1057E-03

Table D-10. Demodulation Performance - QPSK, Power Loop, "Cesium II" Phase Noise (2 Terminals)

{Losses - Soft Decision (3bit), R=1/2, K=7, Viterbi Decoding at BER = 10^{-5}}
*Information bit rate = 1/2 · modulation bit rate.

```
PHASE VARIANCES VS. DATA RATE*
AT OPTIMUM PLL BANDWIDTH:

M-ARY PSK     M= 4

EB/NO= 1.3 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR=  .707107
```

DMR (MOD. BITS/SEC)	BL (HZ)	DB-VAR(TOT) (DB-RAD SQ)	DB-VAR(TH) (DB-RAD SQ)	DB-VAR(DN) (DB-RAD SQ)
19200.	6.794	-26.2339	-30.2431	-28.3444
76800.	10.678	-27.9888	-34.3436	-29.1319
307200.	17.534	-29.6601	-34.2104	-29.1132
1228800.	145.206	-29.1614	-33.4432	-30.8623
4915200.	408.372	-31.2674	-35.7146	-33.2003
19660800.	946.662	-33.1203	-34.2187	-34.3440
78643200.	1372.342	-36.1886	-43.3688	-34.7443

```
DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

 SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR=  .707107
```

DMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
19200.	6.784	.2887E+00	.1135E+00
76800.	10.678	.1924E+00	.4488E-01
307200.	17.534	.1676E+00	.1833E-01
1228800.	145.206	.1472E+00	.4641E-01
4915200.	409.372	.4066E-01	.4257E-01
19660800.	966.662	.5641E-01	.1454E-01
78643200.	1372.342	.6629E-01	.5868E-02

Table D-11. Demodulation Performance - QPSK, Decision Feedback, "Cesium II" Phase Noise (2 Terminals)

{Losses - Soft Decision (3 bit), R=1/2, K=7, Viterbi Decoding at BER = 10^{-5}}
*Information bit rate = 1/2 · modulation bit rate.

PHASE VARIANCES VS. DATA RATE *
AT OPTIMUM PLL BANDWIDTH:

M-ARY PSK M = 4

EB/NO= 3.3 [illegible]

DECISION FEEDBACK IMPLEMENTATION

DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
19200.	8.400	-27.2643	-32.2455	-28.4300
76800.	13.630	-28.5820	-36.2600	-29.3468
307200.	25.860	-28.8837	-34.4443	-29.2750
1228800.	322.360	-30.2107	-34.5831	-32.2115
4915200.	667.681	-32.2777	-37.2424	-33.9215
19660800.	1048.911	-33.7626	-41.4175	-34.5770
78643200.	1703.539	-34.4856	-45.7446	-34.6626

DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

DECISION FEEDBACK IMPLEMENTATION

DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
19200.	8.440	.22475E+00	.71556E-01
76800.	13.630	.16682E+00	.28773E-01
307200.	25.860	.15848E+00	.11683E-01
1228800.	322.360	.11546E+00	.42465E-01
4915200.	627.661	.71455E-01	.22655E-01
19660800.	1048.911	.51285E-01	.84655E-02
78643200.	1703.539	.43722E-01	.35064E-02

Table D-12. Demodulation Performance - BPSK, Power Loop, "Cesium II' Phase Noise (2 Terminals and 1 Equivalent Satellite)

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}
*** Information bit rate = ½ · modulation bit rate**

PHASE VARIANCES VS. DATA RATE *
AT OPTIMUM BL BANDWIDTH:

M-ARY PSK M= 2

EB/N0= 1.3 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR= .707107

BMR (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(FA) (DB-RAD**2)
75.	1.619	-14.7073	-16.5844	-14.2444
300.	2.627	-18.3766	-20.5080	-22.4866
1200.	4.193	-21.6948	-24.4966	-24.9236
4800.	6.736	-24.4463	-28.4543	-26.6454
19200.	10.602	-26.2769	-32.5048	-27.4574
76800.	17.409	-27.0762	-36.3766	-27.6190
307200.	142.520	-27.4788	-32.1918	-29.2691
1228800.	442.842	-29.4412	-33.8485	-31.4266
4915200.	869.117	-31.3264	-37.4557	-32.5410
19660800.	1357.820	-32.4060	-41.5383	-32.9766
78643200.	2230.116	-32.9122	-45.4041	-33.1641

DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .1146 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR= .707107

BMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
75.	1.619	.2134E+00	.1233E+00
300.	2.627	.7544E-01	.4324E-01
1200.	4.193	.3206E-01	.1615E-01
4800.	6.736	.1634E-01	.6307E-02
19200.	10.602	.1055E-01	.2454E-02
76800.	17.409	.8734E-02	.1003E-02
307200.	142.520	.7942E-02	.2641E-02
1228800.	442.842	.4486E-02	.1774E-02
4915200.	869.117	.3229E-02	.7816E-03
19660800.	1357.820	.2514E-02	.3064E-03
78643200.	2230.116	.2235E-02	.1251E-03

Table D-13. Demodulation Performance - QPSK, Power Loop, "Cesium II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}

* Information bit rate = ½ · modulation bit rate

PHASE VARIANCES VS. DATA RATE*
AT OPTIMUM LOOP BANDWIDTH:

M-ARY PSK   M= 4

EB/NO= 1.3 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(OS) (DB-RAD**2)
19200.	7.752	-25.1159	-29.7130	-26.4661
76800.	12.202	-26.5465	-33.7644	-27.5147
307200.	21.046	-27.0076	-37.4176	-27.4214
1228800.	263.249	-28.0076	-32.4653	-29.9342
4915200.	607.177	-30.1136	-34.4672	-31.4623
19660800.	1001.217	-31.7425	-36.7057	-32.7162
78643200.	1547.347	-32.6062	-42.4357	-33.0390

DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
19200.	7.752	.3773E+00	.1296E+00
76800.	12.202	.2657E+00	.5103E-01
307200.	21.046	.2414E+00	.2201E-01
1228800.	263.249	.1420E+00	.6681E-01
4915200.	607.177	.1142E+00	.3468E-01
19660800.	1001.217	.6127E-01	.1036E-01
78643200.	1547.347	.4662E-01	.5320E-02

Table D-14. Demodulation Performance - QPSK, Decision Feedback, "Cesium II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}

*Information bit rate = ½ · modulation bit rate

PHASE VARIANCES VS. DATA RATE *
AT OPTIMUM BL BANDWIDTH:

M-ARY PSK   M= 4

EB/NO= 1.3 DB

DECISION FEEDBACK IMPLEMENTATION

DAMPING FACTOR= .707107

BMR (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
19200.	9.701	-26.0317	-31.7163	-27.3491
76800.	15.575	-27.0643	-35.6609	-27.7300
307200.	119.460	-27.3643	-32.6534	-26.8064
1228800.	425.066	-26.1072	-33.3614	-31.1517
4915200.	739.188	-31.0277	-36.6403	-32.4215
19660800.	1252.282	-32.2506	-40.7106	-32.4166
78643200.	1974.066	-32.8416	-44.7544	-33.1306

DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

DECISION FEEDBACK IMPLEMENTATION

DAMPING FACTOR= .707107

BMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
19200.	9.701	.3024E+00	.8176E-01
76800.	15.575	.2374E+00	.3252E-01
307200.	119.460	.2226E+00	.6293E-01
1228800.	425.066	.1491E+00	.5544E-01
4915200.	779.188	.9582E-01	.2632E-01
19660800.	1252.282	.7230E-01	.1031E-01
78643200.	1974.066	.6310E-01	.4063E-02

Table D-15. Demodulation Performance - BPSK, Power Loop, "Crystal II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

{**Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}**}

* **Information bit rate = ½ · modulation bit rate**

```
PHASE VARIANCES VS. DATA RATE*
AT OPTIMUM LOOP BANDWIDTH:

M-ARY PSK     M= 2

RHO/C= 1.3[illegible]

POWER LOOP IMPLEMENTATION

DAMPING FACTOR=  .707107
```

RMB (MOD. BITS/SEC)	BL (HZ)	PHASE VAR(TOT) (DB-RAD**2)	PHASE VAR(TH) (DB-RAD**2)	PHASE VAR(PN) (DB-RAD**2)
75.	.616	-20.5140	-22.4475	-24.6[illegible]91
300.	.641	-24.2389	-26.3736	-28.3448
1200.	1.1[illegible]1	-27.6131	-30.30[illegible]	-30.[illegible]
4800.	1.751	-30.4054	-34.2114	-32.7444
19200.	3.041	-32.1[illegible]	-37.5[illegible]	-33.6[illegible]
76800.	6.583	-32.5179	-40.6004	-33.2401
307200.	18.657	-31.6227	-42.0484	-32.0[illegible]7
1228800.	54.7[illegible]	-31.6687	-43.4417	-31.6875
4915200.	158.152	-32.0010	-44.1[illegible]	-32.2774
19660800.	465.264	-32.5044	-43.0203	-32.4143
78643200.	1446.783	-32.9264	-45.5871	-33.1501

```
DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:


 SENSITIVITY GAIN (EH) =  10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN <  .11[illegible]

POWER LOOP IMPLEMENTATION

DAMPING FACTOR=  .707107
```

RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
75.	.616	.4[illegible]1E-01	.2[illegible]3[illegible]E-01
300.	.641	.1719E-01	.10[illegible]1E-01
1200.	1.1[illegible]1	.7695E-02	.4[illegible]9[illegible]E-02
4800.	1.751	.4001E-02	.1656E-02
19200.	3.041	.2657E-02	.7000E-03
76800.	6.583	.24[illegible]E-02	.3764E-03
307200.	18.657	.3[illegible]15E-02	.2[illegible]0E-0[illegible]
1228800.	54.75[illegible]	.2[illegible]43E-02	.1556E-03
4915200.	158.152	.2761E-02	.1[illegible]5E-03
19660800.	465.264	.2653E-02	.2156E-03
78643200.	1446.783	.2224E-02	.1119E-03

Table D-16. Demodulation Performance - QPSK, Power Loop, "Crystal II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

{**Losses** - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}

• Information bit rate = ½ · modulation bit rate

[illegible]

AT OPTIMUM [illegible]

[illegible]

[illegible] 1.[illegible]

POWER LOOP [illegible]

DAMPING FACTOR= .707[illegible]

[illegible] (MOD. BITS/SEC)	[illegible]	[illegible]	[illegible]	[illegible]
19200.	2.[illegible]	-31.[illegible]	-3[illegible].[illegible]	-3[illegible].[illegible]
76800.	3.[illegible]	-3[illegible].[illegible]	-3[illegible].[illegible]	-33.[illegible]
307200.	[illegible]	-31.[illegible]	-41.[illegible]	-32.[illegible]
1228800.	[illegible]	-31.47[illegible]	-42.[illegible]	-31.[illegible]
4915200.	[illegible]	-31.7[illegible]	-4[illegible].[illegible]	-32.[illegible]
19660800.	[illegible]	-32.[illegible]	-4[illegible].[illegible]	-32.[illegible]
78643200.	123[illegible].7[illegible]	-3[illegible].[illegible]	-4[illegible].[illegible]	-33.[illegible]

DEMODULATION LOSS VS. [illegible] RATE

AT OPTIMUM [illegible]

SENSITIVITY GAIN [illegible] = [illegible]

DEMODULATION LOSSES [illegible]

ERROR. ARE ACCURATE [illegible]

POWER LOOP [illegible]

DAMPING FACTOR= .7[illegible]

[illegible] (MOD. BITS/SEC)	[illegible]	[illegible]	LOSS(T[illegible]) ([illegible])
19200.	2.[illegible]	.[illegible]	.[illegible]
76800.	7.[illegible]	.71[illegible]	.1[illegible]
307200.	[illegible]	.7[illegible]	.[illegible]
1228800.	[illegible]	.[illegible]	.[illegible]
4915200.	[illegible]	.[illegible]	.[illegible]
19660800.	[illegible]	.7[illegible]	.[illegible]
78643200.	[illegible]	.[illegible]	.[illegible]

Table D-17. Demodulation Performance - QPSK, Decision Feedback, "Crystal II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}
* Information bit rate = ½ · modulation bit rate

PHASE VARIANCES VS. DATA RATE *
AT OPTIMUM PLL BANDWIDTH:

M-ARY PSK M= 4

EB/NO= 1.3 DB

DECISION FEEDBACK IMPLEMENTATION

DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
19200.	2.721	-32.9450	-37.2374	-33.6097
76800.	5.442	-32.4867	-40.2155	-33.7756
307200.	15.118	-32.2429	-41.8306	-32.7484
1228800.	44.146	-31.6921	-43.1424	-32.0110
4915200.	141.081	-31.9379	-44.1721	-32.2156
19660800.	812.452	-32.4012	-42.5894	-32.8344
78643200.	1731.651	-32.8616	-45.3234	-33.1153

DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

DECISION FEEDBACK IMPLEMENTATION

DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
19200.	2.721	.7580E-01	.2244E-01
76800.	5.442	.6245E-01	.1155E-01
307200.	15.118	.7243E-01	.7466E-02
1228800.	44.146	.8222E-01	.5622E-02
4915200.	141.081	.7770E-01	.4646E-02
19660800.	412.452	.6984E-01	.5684E-02
78643200.	1731.651	.6241E-01	.3544E-02

Table D-18. Demodulation Performance - Radiation, Inc. BPSK With Modified HT-MT Phase Noise (2 Terminals)

(Losses - Soft Decision (3 bit), R - 1/2, K = 7, Viterbi Decoding at BER 10^{-5})

LOOP BANDWIDTH AND THE CORRESPONDING PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.26E-10 RAD/HZ   H1= 0 RAD
H2= .01 RAD*HZ   H3= .2 RAD*HZ↑2

M-ARY PSK   M= 2   EB/N0= 1.3 DB

MOD. BIT RATE * B/S	BW (MF) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(PN) DB
1200	175	-8.28419	-8.29215	-35.6588
4800	175	-14.281	-14.3127	-35.6552
19200	175	-20.2072	-20.3333	-35.6406
76800	175	-25.864	-26.3539	-35.5834
307200	175	-30.6057	-32.3745	-35.3612
1228800	175	-33.0648	-38.3951	-34.5711
4915200	175	-32.0938	-44.4157	-32.356
19660800	175	-28.1835	-50.4363	-28.2095
78643200	175	-22.8163	-56.4569	-22.8182

CODING SENSIVITY GAIN= 10 DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN< .116469 DB

MOD. BIT RATE * (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)	Tikhonov Loss (tot) (dB)	Approx. Loss (Th) (dB)
1200	1.93446	.643094	>6	>6
4800	.243479	.160774	1.65	1.65
19200	4.66991E-2	4.01934E-2	<.1	<.1
76800	1.16415E-2	1.00483E-2	.	.
307200	3.8193E-3	2.51009E-3	.	.
1228800	2.15719E-3	6.28022E-4	.	.
4915200	2.70216E-3	1.57005E-4		
19660800	6.70897E-3	3.92514E-5		
78643200	2.42918E-2	9.81284E-6		

* Information bit rate = ½ · modulation bit rate

Table D-19. Demodulation Performance - Radiation Inc. BPSK With Modified HT-MT Phase Noise (2 Terminals + 1 Equivalent Satellite) (Losses - Soft Decision (3 bit), R = 1/2, K = 7, Viterbi Decoding at BER = 10^{-5})

LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS

H0= 1.69E-10 RAD/HZ H1= 0 RAD

H2= .015 RAD*HZ H3= .3 RAD*HZ↑2

M-ARY PSK M= 2 EB/N0= 1.3 DB

MOD. BIT RATE * B/S	BW (MF) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(PN) DB
1200	175	-8.28022	-8.29215	-33.8979
4800	175	-14.2652	-14.3127	-33.8943
19200	175	-20.1455	-20.3333	-33.6798
76800	175	-25.6384	-26.3539	-33.8225
307200	175	-29.934	-32.3745	-33.6002
1228800	175	-31.7504	-38.3951	-32.6101
4915200	175	-30.4185	-44.4157	-30.5951
19660800	175	-26.4312	-50.4363	-26.4465
78643200	175	-21.056	-56.4569	-21.0573

CODING SENSITIVITY GAIN= 10 DB

DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN< .116469 DB

MOD. BIT RATE * (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)	Tikhonov Loss (tot)	Approx. Loss (Th)
1200	1.93741	.643094	>6	>6
4800	.244664	.160774	1.65	1.65
19200	4.74446E-2	4.01934E-2	<.1	<.1
76800	1.22846E-2	1.00483E-2	.	.
307200	4.46661E-3	2.51209E-3	.	.
1228800	2.92652E-3	6.28022E-4	.	.
4915200	3.98954E-3	1.57005E-4	.	
19660800	.010174	3.92514E-5		
78643200	.037632	9.81284E-6		

* Information bit rate = ½ · modulation bit rate

Table D-20. Demodulation Performance - Radiation Inc. BPSK with "Crystal II" Phase Noise (2 Terminals)

(Losses - Soft Decision (3 bit), R = 1/2, K = 7, Viterbi Decoding at BER = 10^{-5})

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 2

EB/NO= 1.3 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
1200.	175.000	-8.2772	-8.2922	-32.9209
4800.	175.000	-14.2495	-14.3128	-32.6462
19200.	175.000	-20.0714	-20.3334	-32.3965
76800.	175.000	-25.2754	-26.3540	-31.8531
307200.	175.000	-28.5923	-32.3745	-30.9474
1228800.	175.000	-30.0330	-38.3952	-30.7174
4915200.	175.000	-30.5170	-44.4157	-30.6977
19660800.	175.000	-30.6504	-50.4363	-30.6963

DEMODULATION LOSS VS. DATA RATE

MINIMUM-FREE DISTANCE OF CODE=10.0
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .116469 DB

				TIKHONOV APPROX.	
RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)	LOSS(TOT) (DB)	LOSS(TH) (DB)
1200.	175.000	.2036E+01	.2024E+01	>6.	>6.
4800.	175.000	.2520E+00	.2471E+00	1.69	1.55
19200.	175.000	.4878E-01	.4559E-01	<.1	<.1
76800.	175.000	.1343E-01	.1039E-01	<.1	<.1
307200.	175.000	.6122E-02	.2533E-02	<.1	<.1
1228800.	175.000	.4369E-02	.6293E-03	<.1	<.1
4915200.	175.000	.3902E-02	.1571E-03	<.1	<.1
19660800.	175.000	.3783E-02	.3926E-04	<.1	<.1

* Information bit rate = 1/2 · modulation bit rate

Table D-21. Demodulation Performance - Radiation Inc. BPSK with "Cesium II" Phase Noise (2 Terminals)

(Losses - Soft Decision (3 bit), R = 1/2, K = 7, Viterbi Decoding at BER = 10^{-5}

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK   M= 2

EB/NO= 1.3 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
1200.	175.000	-8.2697	-8.2922	-31.1509
4800.	175.000	-14.2180	-14.3128	-30.8762
19200.	175.000	-19.9454	-20.3334	-30.6276
76800.	175.000	-24.8201	-26.3540	-30.0845
307200.	175.000	-27.4778	-32.3745	-29.1773
1228800.	175.000	-28.4793	-38.3952	-28.9464
4915200.	175.000	-28.8055	-44.4157	-28.9265
19660800.	175.000	-28.8946	-50.4363	-28.9252

DEMODULATION LOSS VS. DATA RATE

MINIMUM-FREE DISTANCE OF CODE=10.0
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .116469 DB

				TIKHONOV APPROX.	
RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)	LOSS(TOT) (DB)	LOSS(TH) (DB)
1200.	175.000	.2042E+01	.2024E+01	>6.	>6.
4800.	175.000	.2545E+00	.2471E+00	1.76	1.55
19200.	175.000	.5040E-01	.4559E-01	<.1	<.1
76800.	175.000	.1499E-01	.1039E-01	<.1	<.1
307200.	175.000	.7958E-02	.2533E-02	<.1	<.1
1228800.	175.000	.6286E-02	.6293E-03	<.1	<.1
4915200.	175.000	.5823E-02	.1571E-03	<.1	<.1
19660800.	175.000	.5703E-02	.3926E-04	<.1	<.1

* Information bit rate = 1/2 · modulation bit rate

Table D-22. Demodulation Performance - Radiation Inc. BPSK With "Crystal II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

(Losses - Soft Decision (3 bit), R = 1/2, K = 7, Viterbi Decoding at BER = 10^{-5})

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 2

EB/NO= 1.3 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
1200.	175.000	-8.2902	-8.2922	-41.6820
4800.	175.000	-14.3024	-14.3128	-40.5349
19200.	175.000	-20.2775	-20.3334	-39.2157
76800.	175.000	-26.0001	-26.3540	-37.0660
307200.	175.000	-30.3307	-32.3745	-34.5860
1228800.	175.000	-32.7061	-38.3952	-34.0719
4915200.	175.000	-33.6490	-44.4157	-34.0292
19660800.	175.000	-33.9281	-50.4363	-34.0262

DEMODULATION LOSS VS. DATA RATE

MINIMUM-FREE DISTANCE OF CODE=10.0
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .116469 DB

RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH) (DB)
1200.	175.000	.2025E+01	.2024E+01	>6.	>6.
4800.	175.000	.2479E+00	.2471E+00	1.57	1.55
19200.	175.000	.4625E-01	.4559E-01	<.1	<.1
76800.	175.000	.1130E-01	.1039E-01	<.1	<.1
307200.	175.000	.4076E-02	.2533E-02	<.1	<.1
1228800.	175.000	.2346E-02	.6293E-03	<.1	<.1
4915200.	175.000	.1885E-02	.1571E-03	<.1	<.1
19660800.	175.000	.1767E-02	.3926E-04	<.1	<.1

* Information bit rate = 1/2 · modulation bit rate

Table D-23. Demodulation Performance - Radiation Inc. BPSK with "Cesium II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

(Losses - Soft Decision (3 bit), R = 1/2, K = 7, Viterbi Decoding at BER = 10^{-5})

PHASE VARIANCES VS. DATA RATE*

M-ARY PSK M= 2

EB/NO= 1.3 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2
1200.	175.000	-8.2892	-8.2922	-39.9120
4800.	175.000	-14.2972	-14.3128	-38.7649
19200.	175.000	-20.2498	-20.3334	-37.4509
76800.	175.000	-25.8330	-26.3540	-35.3008
307200.	175.000	-29.5792	-32.3745	-32.8157
1228800.	175.000	-31.3454	-38.3952	-32.2996
4915200.	175.000	-32.0003	-44.4157	-32.2567
19660800.	175.000	-32.1883	-50.4363	-32.2538

DEMODULATION LOSS VS. DATA RATE

MINIMUM-FREE DISTANCE OF CODE=10.0
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .116469 DB

RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH) (DB)
1200.	175.000	.2026E+01	.2024E+01	>6.	>6.
4800.	175.000	.2483E+00	.2471E+00	1.59	1.55
19200.	175.000	.4658E-01	.4559E-01	<.1	<.1
76800.	175.000	.1176E-01	.1039E-01	<.1	<.1
307200.	175.000	.4858E-02	.2533E-02	<.1	<.1
1228800.	175.000	.3218E-02	.6293E-03	<.1	<.1
4915200.	175.000	.2763E-02	.1571E-03	<.1	<.1
19660800.	175.000	.2645E-02	.3926E-04	<.1	<.1

* Information bit rate = 1/2 · modulation bit rate

Table D-24. Demodulation Performance - Radiation Inc. BPSK with "Cesium II" Phase Noise (2 Terminals and 1 Equivalent Satellite), PLL Damping ξ 1.0

(Losses - Soft Decision (3 bit), R = 1/2, K = 7, Viterbi Decoding at BER = 10^{-5}

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 2

EB/NO= 1.3 DB

POWER LOOP IMPLEMENTATION

DAMPING FACTOR= 1.000000

RMB (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
1200.	175.000	-8.2669	-8.2922	-30.6414
4800.	175.000	-14.2070	-14.3128	-30.3932
19200.	175.000	-19.9043	-20.3334	-30.1698
76800.	175.000	-24.6951	-26.3540	-29.6780
307200.	175.000	-27.2500	-32.3745	-28.8445
1228800.	175.000	-28.1944	-38.3952	-28.6303
4915200.	175.000	-28.4991	-44.4157	-28.6118
19660800.	175.000	-28.5821	-50.4363	-28.6105

DEMODULATION LOSS VS. DATA RATE

MINIMUM-FREE DISTANCE OF CODE=10.0
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .116469 DB

RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)	TIKHONOV APPROX. LOSS(TOT) (DB)	TIKHONOV APPROX. LOSS(TH) (DB)
1200.	175.000	.2044E+01	.2024E+01	>6.	>6.
4800.	175.000	.2553E+00	.2471E+00	1.78	1.55
19200.	175.000	.5094E-01	.4559E-01	<.1	<.1
76800.	175.000	.1545E-01	.1039E-01	<.1	<.1
307200.	175.000	.8398E-02	.2533E-02	<.1	<.1
1228800.	175.000	.6722E-02	.6293E-03	<.1	<.1
4915200.	175.000	.6257E-02	.1571E-03	<.1	<.1
19660800.	175.000	.6136E-02	.3926E-04	<.1	<.1

* Information bit rate = 1/2 · modulation bit rate

Table D-25. Demodulation Performance - Raytheon Inc. BPSK TDMA With HT-MT Mod Phase Noise (2 Terminals)

(Losses - Soft Decision (3 bit), R=1/2, K=7, Viterbi Decoding @ BER=10^{-5})

```
AUXILLIARY CARRIER SYSTEM
TDMA DUTY FACTOR =   .001



        LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.26E-10  RAD/HZ                 H1= 0  RAD
H2= .01  RAD*HZ                      H3= .2  RAD*HZ↑2

M-ARY PSK    M= 2                    EB/N0= 1.3  DB
```

Mod. BIT RATE * B/S	B-LOOP(NF) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(PN) DB
307200	100	-6.1639	-6.17421	-32.4127
1228800	100	-12.1496	-12.1949	-31.9441
4915200	100	-17.9735	-18.2154	-30.[illegible]
19660800	100	-22.5465	-24.236	-27.[illegible]
78643200	100	-21.9034	-30.2566	-22.[illegible]

```
CODING SENSITIVITY GAIN =  10  DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN<  .116469  DB
```

Mod. BIT RATE * (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)	Tikhonov Approx Loss(Tot) (DB)	Tikhonov Approx Loss (TH) (DB)
307200	4.47517	1.04729	>6	>6
1228800	.482124	.261823	>6	>6
4915200	9.40925E-2	6.54559E-2	<.1	<.1
19660800	2.59578E-2	.016364	.	.
78643200	3.[illegible]4365E-2	4.09099E-3	.	.

* Information bit rate = 1/2 · modulation bit rate

Table D-26. Demodulation Performance - Raytheon Inc. QPSK TDMA With HT-MT Mod Phase Noise (2 Terminals)

(Losses - Soft Decision (3 bit), R=1/2, K=7, Viterbi Decoding @ BER=10^{-5})

```
AUXILIARY CARRIER SYSTEM
TDMA DUTY FACTOR =   .001

        LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.26E-10  RAD/HZ                H1= 0  RAD
H2= .01  RAD*HZ                     H3= .2  RAD*HZ↑2

M-ARY PSK    M= 4                   EB/N0= 1.3  DB
```

Mod. BIT RATE * B/S	BW- (YE) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(FN) DB
307200	100	-6.164[illegible]	-6.17421	-32.48[illegible]
1228800	100	-12.1503	-12.19[illegible]	-32.[illegible]
4915200	100	-18.0153	-18.2154	-31.[illegible]21
19660800	100	-23.059	-24.23[illegible]	-29.3[illegible]
78643200	100	-24.0074	-30.256[illegible]	-2[illegible].18[illegible]

```
CODING SENSITIVITY  AIN =  10  DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN<  .4  DB
```

Mod. BIT RATE * (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)	Tikhonov Approx. Loss(Tot) (DB)	Tikhonov Approx. Loss (TH) (DB)
307200	25.9[illegible]59	25.8933	> 6	> 6
1228800	7.13036	7.11255	> 6	> 6
4915200	1.90[illegible]93	1.81809	> 6	> 6
19660800	.59[illegible]9[illegible]	.[illegible]5701[illegible]	1.0	.65
78643200	.481673	.114[illegible]11	.95	< .15

* Information bit rate = 1/2 modulation bit rate

Table D-27. Demodulation Performance - Raytheon Inc. BPSK TDMA With "Cesium II" Phase Noise (2 Terminals)

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}

PHASE VARIANCES VS. DATA RATE*

M-ARY PSK [illegible]

EB/N0 = 1.[illegible]

AUXILIARY CARRIER SYSTEM
TIME DUTY FACTOR = .[illegible]
RANGING FACTOR = .707107

DATA (MOD. BITS/SEC)	(%)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
307200.	100.000	-6.1541	-6.1742	-29.9737
1228800.	100.000	-12.1154	-12.1948	-29.7890
4915200.	100.000	-17.4241	-18.2154	-29.7730
19660800.	100.000	-23.1655	-24.2360	-29.7719
78643200.	100.000	-26.9670	-30.2566	-29.7716

DEMODULATION LOSS VS. DATA RATE

SENSITIVITY DATA (DB) = 1[illegible].00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. [illegible] ACCURATE [illegible] < .11[illegible] DB

RANGING FACTOR = .707107

DATA (MOD. BITS/SEC)	(%)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	100.000	.4449E+01	.4456E+01
1228800.	100.000	.4559E+00	.4749E+00
4915200.	100.000	.5527E-01	.7877E-01
19660800.	100.000	.2230E-01	.1720E-01
78643200.	100.000	.5590E-02	.4143E-02

* Information bit rate = ½ · modulation bit rate

Table D-28. Demodulation Performance - Raytheon Inc. BPSK TDMA With "Cesium II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 2

EB/NO= 1.3 DB

AUXILIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .00100000
DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
307200.	100.000	-6.1471	-6.1762	-28.2136
1228800.	100.000	-12.0827	-12.1948	-28.0181
4915200.	100.000	-17.7817	-18.2154	-28.0021
19660800.	100.000	-22.7124	-24.2360	-28.0011
78643200.	100.000	-25.9735	-30.2566	-28.0007

DEMODULATION LOSS VS. DATA RATE

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .11646 DB

DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	100.000	.4506E+01	.4456E+01
1228800.	100.000	.4930E+00	.4749E+00
4915200.	100.000	.8859E-01	.7877E-01
19660800.	100.000	.2492E-01	.1720E-01
78643200.	100.000	.1134E-01	.4143E-02

* Information bit rate = $\frac{1}{2}$ · modulation bit rate

{Losses - Soft Decision (3 bit), R=$\frac{1}{2}$, K=7, Viterbi Decoding @ BER = 10^{-5}}

Table D-29. Demodulation Performance - Raytheon Inc. QPSK
TDMA With "Cesium II" Phase Noise (2 Terminals)

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 4

EB/NO= 1.3

AUXILIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .0010000
DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	W (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
307200.	100.000	-6.1574	-6.1742	-30.2971
1228800.	100.000	-12.1297	-12.1948	-29.8338
4915200.	100.000	-17.9224	-18.2154	-29.7764
19660800.	100.000	-23.1656	-24.2360	-29.7721
78643200.	100.000	-26.9471	-30.2566	-29.7718
314572800.	100.000	-28.8960	-36.2772	-29.7713

DEMODULATION LOSS VS. DATA RATE

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	W (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	100.000	.2544E+02	.2549E+02
1228800.	100.000	.7231E+01	.7113E+01
4915200.	100.000	.1944E+01	.1818E+01
19660800.	100.000	.5845E+00	.4570E+00
78643200.	100.000	.2422E+00	.1144E+00
314572800.	100.000	.1565E+00	.2661E-01

* Information bit rate = $\frac{1}{2}$ · modulation bit rate

{Losses - Soft Decision (3 bit), R=$\frac{1}{2}$, K=7, Viterbi Decoding @ BER=10^{-5}}

Table D-30. Demodulation Performance - Raytheon Inc. QPSK TDMA With "Cesium II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 4

EB/NO= 1.3 DB

AUXILLIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .00100000
DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	[illegible]
307200.	100.000	-6.1490	-6.1742	[illegible]
1228800.	100.000	-12.0838	-12.1444	[illegible]
4915200.	100.000	-17.7820	-18.2154	[illegible]
19660800.	100.000	-22.7125	-24.2360	[illegible]
78643200.	100.000	-25.9736	-30.2666	[illegible]
314572800.	100.000	-27.3983	-36.2772	[illegible]

DEMODULATION LOSS VS. DATA RATE

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	100.000	.2602E+02	.2549E+02
1228800.	100.000	.7291E+01	.7113E+01
4915200.	100.000	.2007E+01	.1818E+01
19660800.	100.000	.6486E+00	.4570E+00
78643200.	100.000	.3065E+00	.1144E+00
314572800.	100.000	.2209E+00	.2861E-01

* Information bit rate = ½ · modulation bit rate

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}

Table D-31. Demodulation Performance - Raytheon Inc. BPSK TDMA With "Crystal II" Phase Noise (2 Terminals)

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 2

EB/NO= 1.3 DB

AUXILIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .00100000
DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	[illegible] (%)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
307200.	100.000	-6.1677	-6.1742	-34.4105
1228800.	100.000	-12.1657	-12.1948	-33.9155
4915200.	100.000	-18.0990	-18.2154	-33.8744
19660800.	100.000	-2[illegible].7677	-24.2360	-33.8715
78643200.	100.000	-2[illegible].[illegible]	-30.2566	-33.8708

DEMODULATION LOSS VS. DATA RATE

SENSITIVITY GAIN (DB) = 1[illegible].00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE TO < .11[illegible] DB

DAMPING FACTOR= .7[illegible]71[illegible]7

DMR (MOD. BITS/SEC)	[illegible] (%)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	[illegible]	.4[illegible]E+01	.4656E+01
1228800.	[illegible]	.47[illegible]E+00	.3749E+00
4915200.	100.000	.[illegible]12[illegible]E-01	.7677E-01
19660800.	100.000	.1[illegible]17E-01	.1720E-01
78643200.	100.000	.5[illegible]7[illegible]E-02	.4143E-02

* **Information bit rate = ½ · modulation bit rate**

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}

Table D-32. Demodulation Performance - Raytheon Inc. BPSK TDMA With "Crystal II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 2

EB/NO= 1.3 DB

AUXILIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .00100000
DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(OS) (DB-RAD**2)
307200.	100.000	-6.1644	-6.1742	-32.64[illegible]
1228800.	100.000	-12.1511	-12.1448	-32.1433
4915200.	100.000	-18.0415	-18.2154	-32.1020
19660800.	100.000	-23.5761	-24.2360	-32.1662
78643200.	100.000	-28.0703	-30.2566	-32.0444

DEMODULATION LOSS VS. DATA RATE

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .116469 DB

DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	100.000	.4474E+01	.4456E+01
1228800.	100.000	.4819E+00	.4749E+00
4915200.	100.000	.9256E-01	.7677E-01
19660800.	100.000	.2017E-01	.1720E-01
78643200.	100.000	.6910E-02	.4143E-02

* Information bit rate = ½ · modulation bit rate

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}

Table D-33. Demodulation Performance - Raytheon Inc. QPSK TDMA With "Crystal II" Phase Noise (2 Terminals)

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 4

EB/NO= 1.3 DB

AUXILLIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .00100000
DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BI (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
307200.	100.000	-6.1660	-6.1742	-35.3758
1228800.	100.000	-12.1665	-12.1948	-34.0325
4915200.	100.000	-16.0962	-16.2154	-33.8630
19660800.	100.000	-22.7417	-24.2360	-33.8722
78643200.	100.000	-26.6670	-30.2566	-33.8712
314572800.	100.000	-31.4946	-36.2772	-33.8700

DEMODULATION LOSS VS. DATA RATE *

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < [illegible]

DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BI (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	100.000	.2542E+02	.2549E+02
1228800.	100.000	.7158E+01	.7113E+01
4915200.	100.000	.1667E+01	.1618E+01
19660800.	100.000	.5066E+00	.4570E+00
78643200.	100.000	.1642E+00	.1144E+00
314572800.	100.000	.7640E-01	.2661E-01

* Information bit rate = ½ · modulation bit rate

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}

Table D-34. Demodulation Performance - Raytheon Inc. QPSK TDMA With "Crystal II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

PHASE VARIANCES VS. DATA RATE *

M-ARY PSK M= 4

EB/NO= 1.3 DB

AUXILLIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .00100000
DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	[illegible]
307200.	100.000	-6.1664	-6.1742	-33.[illegible]
1228800.	100.000	-12.1522	-12.1448	-32.2[illegible]7
4915200.	100.000	-18.0418	-18.2154	-32.11[illegible]7
19660800.	100.000	-23.5782	-24.2360	-32.[illegible]
78643200.	100.000	-28.0705	-30.2566	-32.[illegible]
314572800.	100.000	-30.6926	-36.2772	-32.[illegible]7[illegible]

DEMODULATION LOSS VS. DATA RATE

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	100.000	.2593E+02	.258[illegible]E+02
1228800.	100.000	.7180E+01	.7113E+01
4915200.	100.000	.1892E+01	.1818E+01
19660800.	100.000	.5316E+00	.4570E+00
78643200.	100.000	.1892E+00	.1144E+00
314572800.	100.000	.1035E+00	.2661E-01

* Information bit rate = ½ · modulation bit rate

{Losses - Soft Decision (3 bit), R=½, K=7, Viterbi Decoding @ BER=10^{-5}}

Table D-35. Demodulation Performance - Raytheon Inc. BPSK TDMA With HT-MT Mod Phase Noise (2 Terminals)

AUXILLIARY CARRIER SYSTEM*
TDMA DUTY FACTOR = .001

OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.26E-10 RAD/HZ   H1= 0 RAD
H2= .01 RAD*HZ   H3= .2 RAD*HZ↑2

M-ARY PSK M= 2   EB/N0= 1.3 DB

BIT RATE B/S	BW-OPT(MF) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(FO) DB
307200	11.5185	-13.4751	-15.5603	-17.6624
1228800	18.2834	-17.2938	-19.5742	-21.1819
4915200	29.0166	-20.8878	-23.589	-24.2309
19660800	46.0189	-23.3268	-27.6067	-25.356
78643200	72.7784	-21.8728	-31.6366	-22.3574

CODING SENSITIVITY GAIN = 10 DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN< .116469 DB

BIT RATE (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200	.313139	.120632
1228800	.101287	4.78703E-2
4915200	.039266	.018993
19660800	2.14397E-2	7.53051E-3
78643200	3.06686E-2	2.97736E-3

*Information bit rate $\frac{1}{2}$ modulation bit rate

(Losses - Soft Decision (3 bit), R $\frac{1}{2}$, K 7, Viterbi Decoding @ BER 10^{-5})

Table D-36. Demodulation Performance - Raytheon Inc. BPSK TDMA With HT-MT Mod Phase Noise (2 Terminals and 1 Equivalent Satellite)

AUXILLIARY CARRIER SYSTEM*
TDMA DUTY FACTOR = .001

OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.89E-10 RAD/HZ   H1= 0 RAD
H2= .015 RAD*HZ   H3= .3 RAD*HZ↑2

M-ARY PSK M= 2   EB/N0= 1.3 DB

BIT RATE B/S	BW-OPT(RF) HZ	PH-VAR(TOT) DB	PH-VAR(TH) DB	PH-VAR(RJ) DB
307200	13.1853	-12.8428	-14.9733	-16.9576
1228800	20.9285	-16.6336	-18.9874	-20.4177
4915200	33.2166	-20.1537	-23.0026	-23.3316
19660800	52.6462	-22.2951	-27.0223	-24.0781
78643200	83.0975	-20.3229	-31.0607	-20.7057

CODING SENSITIVITY GAIN = 10 DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN< .116469 DB

BIT RATE (B/S)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200	.383632	.138088
1228800	.121807	5.47958E-2
4915200	4.73454E-2	2.17383E-2
19660800	2.76195E-2	.008615
78643200	4.53362E-2	3.39951E-3

*Information bit rate $\frac{1}{2}$ modulation bit rate

(Losses - Soft Decision (3 bit), R $\frac{1}{2}$, K 7, Viterbi Decoding @ BER 10^{-5})

Table D-37. Demodulation Performance - Raytheon Inc. QPSK TDMA With HT-MT Mod Phase Noise (2 Terminals)

```
AUXILIARY CARRIER SYSTEM*
TDMA DUTY FACTOR =  .001



OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.26E-10  RAD/HZ            H1= 0  RAD
H2= .01  RAD*HZ                 H3= .2  RAD*HZ↑2

M-ARY FSK   M= 4                EB/N0= 1.3  DB

BIT RATE       BW-OPT(MR)       PH-VAR(TOT)      PH-VAR(TH)       PH-VAR(FQ)
  B/S             HZ               DB               DB               DB
 307200         11.5185          -13.476          -15.5663         -17.6648
 1228800        18.2834          -17.3028         -19.5742         -21.2041
 4915200        29.0166          -20.9711         -23.589          -24.4128
 19660800       46.0189          -23.9476         -27.6067         -26.3939
 78643200       72.7784          -23.9579         -31.6366         -24.7766

CODING SENSITIVITY GAIN =  10  DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN<  .4  DB


BIT RATE       LOSS(TOT)        LOSS(TH)
 (B/S)           (DB)             (DB)

 307200         5.33603          3.32998
 1228800        2.23938          1.33225
 4915200        .967255          .53029
 19660800       .488317          .210522
 78643200       .487181          8.32767E-2
```

*Information bit rate $\frac{1}{2}$ modulation bit rate

(Losses - Soft Decision (3 bit), R $\frac{1}{2}$, K 7, Viterbi Decoding @ BER 10^{-5}.

Table D-38. Demodulation Performance - Raytheon Inc. QPSK TDMA With HT-MT Mod Phase Noise (2 Terminals and 1 Equivalent Satellite)

AUXILIARY CARRIER SYSTEM*
TDMA DUTY FACTOR = .001

OPTIMUM LOOP BANDWIDTH AND THE CORRESPONDING
PHASE NOISE VARIANCE

OSCILLATOR SPECTRAL CHARACTERISTICS
H0= 1.89E-10 RAD/HZ   H1= 0 RAD
H2= .015 RAD*HZ   H3= .3 RAD*HZ↑2

M-ARY PSK M= 4   EB/N0= 1.3 DB

BIT RATE B/S	BW-OPT(RF) HZ	PH-VAR(ICT) DB	PH-VAR(TH) DB	PH-VAR(FX) DB
307200	13.1853	-12.844	-14.9733	-16.9007
1228800	20.9265	-16.6453	-18.9874	-20.4455
4915200	33.2106	-20.2595	-23.0026	-23.5544
19660800	52.6462	-23.0398	-27.0223	-25.2562
78643200	83.0975	-22.5433	-31.0007	-23.2018

CODING SENSITIVITY GAIN = 10 DB
DEMODULATION LOSSES USING GAUSSIAN
APPROX ARE ACCURATE WHEN< .4 DB

BIT RATE (B/S)	LOSS(ICT) (DB)	LOSS(TH) (DB)
307200	6.15046	3.80434
1228800	2.6016	1.52381
4915200	1.13868	.606753
19660800	.601599	.240811
78643200	.674264	9.50798E-2

*Information bit rate $\frac{1}{2}$ modulation bit rate

(Losses - Soft Decision (3 bit), R $\frac{1}{2}$, K 7, Viterbi Decoding @ BER 10^{-5}

Table D-39. Demodulation Performance - Raytheon Inc. BPSK
TDMA with "Cesium II" Phase Noise (2 Terminals)

Phase Variances vs. Data Rate*
at Optimum PLL Bandwidth:

[illegible]

[illegible]

[illegible]
TDMA [illegible]
DAMPING FACTOR = [illegible]

RATE (MOD. BITS/SEC)	[illegible]	PHVAR(TOT)	PHVAR(TH)	PHVAR(PN)
307200.	[illegible]	-19.7[illegible]17	-22.0777	-23.5746
1228800.	4.147	-2[illegible].[illegible]27	-26.0176	-26.0052
4915200.	[illegible]	[illegible]	-30.0290	-27.5214
19660800.	[illegible]	-27.2811	-34.0795	-28.2944
78643200.	[illegible]	[illegible]	-37.9711	-28.6518

DEMODULATION [illegible]
AT OPTIMUM [illegible]:

[illegible]
DEMODULATION [illegible]
[illegible]

DAMPING FACTOR = [illegible]

RATE (MOD. BITS/SEC)	[illegible]	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	[illegible]	[illegible]E-01	.2415E-01
1228800.	4.147	[illegible]E-01	.1122E-01
4915200.	[illegible]	.1[illegible]E-01	.4369E-02
19660800.	[illegible]	[illegible]E-02	.1705E-02
78643200.	[illegible]	[illegible]E-02	.5439E-03

*Information bit rate = ½ · modulation bit rate

{Losses - Soft Decision (3 bit), R = ½, K - 7, Viterbi Decoding @ BER = 10^{-5}}

Table D-40. Demodulation Performance - Raytheon Inc. BPSK TDMA with "Cesium II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

Phase Variances vs. Data Rate*
at Optimum PLL Bandwidth:

M-ARY PSK   M= 2

EB/NO= 1.3 [illegible]

AUXILIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .00100000
DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
307200.	2.935	-16.9985	-21.4985	-22.5871
1228800.	4.688	-22.0785	-25.4648	-24.7259
4915200.	7.526	-24.4086	-29.4448	-26.0406
19660800.	11.846	-25.8477	-33.5003	-26.6656
78643200.	20.138	-26.5949	-37.2164	-26.9647

DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .118689 DB

DAMPING FACTOR= .707107

RMB (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	2.935	.6394E-01	.3367E-01
1228800.	4.688	.2914E-01	.1274E-01
4915200.	7.526	.1650E-01	.5002E-02
19660800.	11.846	.1169E-01	.1950E-02
78643200.	20.138	.4794E-02	.4260E-03

*Information bit rate = $\frac{1}{2}$ · modulation bit rate

‡Losses - Soft Decision (3 bit), R = $\frac{1}{2}$, K 7, Viterbi Decoding @ BER = 10^{-5}‡

Table D-41. Demodulation Performance - Raytheon Inc. QPSK TDMA with "Cesium II" Phase Noise (2 Terminals)

Phase Variances vs. Data Rate*
at Optimum PLL Bandwidth:

```
M-ARY PSK     M= 4

EB/N0= 1.3 DB
AUXILLIARY CARRIER SYSTEM
TDMA DUTY FACTOR =  .0010000
DAMPING FACTOR=  .707107
```

DMR (MOD. BITS/SEC)	SL (DB)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
307200.	2.568	-14.7814	-22.1777	-23.6466
1228800.	4.106	-23.0122	-26.0606	-25.9848
4915200.	6.588	-25.5876	-30.0290	-27.5234
19660800.	10.367	-27.2813	-34.0799	-28.2996
78643200.	16.426	-28.1975	-37.4711	-28.6809
314572800.	167.570	-28.9820	-34.0353	-30.6085

```
DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

SENSITIVITY GAIN (K) =  10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN <  .400000 DB

DAMPING FACTOR=  .707107
```

DMR (MOD. BITS/SEC)	SL (DB)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	2.568	.1271E+01	.7503E+00
1228800.	4.106	.6054E+00	.3004E+00
4915200.	6.588	.3350E+00	.1206E+00
19660800.	10.367	.2264E+00	.4746E-01
78643200.	16.426	.1834E+00	.1937E-01
314572800.	167.570	.1534E+00	.4794E-01

*Information bit rate = $\frac{1}{2}$ · modulation bit rate

{Losses - Soft Decision (3 bit), R = $\frac{1}{2}$, K = 7, Viterbi Decoding @ BER = 10^{-5}}

Table D-43. Demodulation Performance - Raytheon Inc. BPSK TDMA with "Crystal II" Phase Noise (2 Terminals)

Phase Variances vs. Data Rate*
at Optimum PLL Bandwidth

M-ARY PSK M= 2

FEC/R= 1.[illegible]

ARTIFICIAL CARRIER SYSTEM
TIME DUTY FACTOR = .0010000
CARRIER FACTOR = .707107

DATA (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
307200.	.666	-25.3186	-27.4493	-28.7448
1228800.	1.081	-28.1815	-31.8577	-30.6146
4915200.	1.749	-30.3122	-35.7871	-31.7598
19660800.	2.966	-31.5270	-39.5171	-32.3474
78643200.	6.242	-32.3869	-42.3033	-32.6561

DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .1164[illegible] DB

CARRIER FACTOR = .707107

DATA (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	.666	.1326E-01	.7110E-02
1228800.	1.081	.6732E-02	.2854E-02
4915200.	1.749	.4040E-02	.1149E-02
19660800.	2.966	.3012E-02	.4658E-03
78643200.	6.242	.2623E-02	.2556E-03

*Information bit rate = $\frac{1}{2}$ · modulation bit rate

{Losses - Soft Decision (3 bit), R = $\frac{1}{2}$, K = 7, Viterbi Decoding @ BER = 10^{-5}}

Table D-45. Demodulation Performance - Raytheon Inc. QPSK TDMA with "Crystal II" Phase Noise (2 Terminals)

Phase Variances vs. Data Rate*
at Optimum PLL Bandwidth

M-ARY PSK   M= 4

EB/NO= 1.3 [illegible]

AUXILLIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .00100000
DAMPING FACTOR= .707107

BMR (MOD. BITS/SEC)	BL (HZ)	PH-VAR(TOT) (DB-RAD**2)	PH-VAR(TH) (DB-RAD**2)	PH-VAR(PN) (DB-RAD**2)
307200.	.664	-25.4266	-27.4493	-28.9662
1228800.	1.081	-28.2125	-31.8577	-30.6669
4915200.	1.749	-30.3160	-35.7871	-31.7651
19660800.	2.964	-31.6274	-34.5171	-32.3979
78643200.	6.242	-32.3492	-42.3033	-32.8564
314572800.	17.692	-32.9358	-43.7493	-33.3072

DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH:

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

DAMPING FACTOR= .707107

BMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	.664	.3476E+00	.1446E+00
1228800.	1.081	.1831E+00	.7914E-01
4915200.	1.749	.1124E+00	.3203E-01
19660800.	2.964	.8345E-01	.1357E-01
78643200.	6.242	.7003E-01	.7145E-02
314572800.	17.692	.6175E-01	.5063E-02

*Information bit rate = ½ · modulation bit rate

{Losses - Soft Decision (3 bit), R = ½, K = 7, Viterbi Decoding @ BER = 10^{-5}}

Table D-46. Demodulation Performance - Raytheon Inc. QPSK TDMA with "Crystal II" Phase Noise (2 Terminals and 1 Equivalent Satellite)

Phase Variances vs. Data Rate*
at Optimum PLL Bandwidth:

M-ARY PSK   N = 4

EB/NO= 1.3 -

AUXILLIARY CARRIER SYSTEM
TDMA DUTY FACTOR = .010000
DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	PHSVAR(TOT) (DB--RAD**2)	PHSVAR(TH) (DB--RAD**2)	PHSVAR(SP) (DB--RAD**2)
307200.	.76[illegible]	-26.6254	-27.4405	-27.9466
1228800.	1.245	-27.1665	-31.2424	-29.3246
4915200.	2.015	-29.6104	-35.1723	-30.2143
19660800.	3.565	-30.1210	-38.7151	-30.7658
78643200.	4.240	-31.7413	-41.0712	-31.2188
314572800.	24.260	-31.3062	-42.4283	-31.6531

DEMODULATION LOSS VS. DATA RATE
AT OPTIMUM BANDWIDTH: .

SENSITIVITY GAIN (DB) = 10.00
DEMODULATION LOSSES USING GAUSSIAN
APPROX. ARE ACCURATE WHEN < .400000 DB

DAMPING FACTOR= .707107

DMR (MOD. BITS/SEC)	BL (HZ)	LOSS(TOT) (DB)	LOSS(TH) (DB)
307200.	.763	.4174E+00	.2235E+00
1228800.	1.245	.2424E+00	.9113E-01
4915200.	2.015	.1524E+00	.3690E-01
19660800.	3.565	.1141E+00	.1632E-01
78643200.	4.240	.1012E+00	.9468E-02
314572800.	24.260	.8640E-01	.6942E-02

*Information bit rate = ½ · modulation bit rate

{Losses - Soft Decision (3 bit), R = ½, K = 7, Viterbi Decoding @ BER = 10^{-5}}

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